• three periods: $T = 0, 1, 2$
• a single good
• a continuum of agents with measure 1
• Each agent is endowed with 1 unit of the good in period 0.
The Model: Asset Return

\[ T = 0 \quad T = 1 \quad T = 2 \]

\[
\begin{array}{c}
-1 \\
\end{array}
\quad \quad \quad
\begin{cases}
0 \\
1 \\
R \\
0
\end{cases}
\]
The Model: Preferences

- In period 0, all agents are identical.
- In period 1, some agents become “patient” and others become “impatient”. (private information)

\[
\begin{align*}
    u(c_1) & \quad \text{if impatient} \\
    u(c_2) & \quad \text{if patient}
\end{align*}
\]
- The probability of being impatient is \(\lambda\) for each agent in period 0.
Autarky

- autarky:
  - utility of the impatient in period 1: $u(1)$
  - utility of the patient in period 2: $u(R)$
  - expected utility in period 0: $\lambda u(1) + (1 - \lambda) u(R)$

- $1 < R$
  - “insurance” against the liquidity shock is desirable.
Banks offers demand deposit contract \( (d_1, d_2) \).

Agents

- make deposits in period 0.
- withdraw \( d_1 \) in period 1.
- or withdraw \( d_2 \) in period 2.

free-entry banking sector: \( (d_1, d_2) \) maximizes the depositor’s expected utility.
Optimal Deposit Contract

\[
\max_{d_1, d_2} \lambda u(d_1) + (1 - \lambda) u(d_2)
\]

s.t. \( (1 - \lambda)d_2 \leq (1 - \lambda d_1) R \) (RC)

withdrawals in period 2 \quad resources in period 2

\( d_1 \leq d_2 \) (IC)
Optimal Deposit Contract:

\[(1 - \lambda)d_2 = (1 - \lambda d_1)R\]

\[slope = \frac{\lambda}{1 - \lambda}R\]
Optimal Deposit Contract:

\[ \lambda u(d_1) + (1 - \lambda) u(d_2) = \text{const} \]

\[ \text{slope} = -\frac{\lambda}{1 - \lambda} \frac{u'(d_1^*)}{u'(d_2^*)} \]

\[ (1 - \lambda) d_2 = (1 - \lambda d_1) R \]

\[ \text{slope} = -\frac{\lambda}{1 - \lambda} R \]

\[ \frac{\lambda}{1 - \lambda} \frac{u'(d_1^*)}{u'(d_2^*)} = \frac{\lambda}{1 - \lambda} R \]

\[ \boxed{\text{MRS}} = \boxed{\text{MRT}} \]
What do banks do?

- $u'(d_1^*) / u'(d_2^*) = R$
- $u'' < 0 \Rightarrow d_1^* < d_2^*$
- CRRA: $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$
  - $u'(c) = c^{-\gamma} \Rightarrow u'(d_1) / u'(d_2) = (d_2 / d_1)^\gamma$
  - if $\gamma = 1 \Rightarrow d_1^* = 1, d_2^* = R$
  - if $\gamma > 1 \Rightarrow 1 < d_1^* < d_2^* < R$
Why do bank runs occur?

- \( \gamma > 1 \implies 1 < d_1^* < d_2^* < R \)
- IC: \( d_1 \leq d_2 \)
- Expectation: Only the impatient depositors withdraw in period 1.
- A patient depositor can \( \begin{cases} 
\text{get } d_2^* & \text{if he withdraws in period 2} \\
\text{get } d_1^* & \text{if he withdraws in period 1}
\end{cases} \)
Why do bank runs occur?

- $\gamma > 1 \implies 1 < d_1^* < d_2^* < R$

- Expectation: All other depositors demand withdraw in period 1.

- A patient depositor can

  \[ \begin{aligned}
  &\text{get nothing} & \text{if he withdraws in period 2} \\
  &\text{get } d_1^* \text{ w.p. } (1/d_1^*) & \text{if he withdraws in period 1}
  \end{aligned} \]