Information Frictions and the Political Economy of Price-Level Volatility

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Based on Joint Work With

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• Goal: Build a model in which one can explore the efficiency (upside) versus the volatility (downside) of the financial economy.

• Compare models of money taxation with corresponding models of costly commodity taxation.


• Costly commodity taxation: We employ (with apology to Wallace, Kiyotaki and Wright, ...) the iceberg short-cut borrowed from international trade.

• Cost of commodity taxation is melting of transferred chocolates.

• Cost of money taxation is potential price-level volatility.
• Adam Smith (1776) on money taxes:
  "A prince who should enact that a certain proportion of his taxes should be paid in a paper money of a certain kind might thereby give a certain value to this paper money."


• Money is the root of excess economic volatility.
Heart of the present paper is the observation: Excess volatility is “typically” associated with allocations that are not Pareto-optimal.

But volatile equilibrium allocations are not typically Pareto superior to the non-volatile equilibrium allocations.

There are winners and losers from excess volatility. See also Goenka and Prechac (2006), Kang (2012 a,b).

Political Economy:

- The planner: Which regime (real taxation or monetary taxation) will he choose? Which tax rates will he choose?
- Voters: Which regime? Which taxes?

• Closely related: Cass and Shell (1983) on sunspots.

Basic Economy

- Single commodity in each state and three consumers, \( h = 1, 2, 3 \).
- Commodity space \( X_h = \mathbb{R}^{++} \).
- Consumers preferences identical: differ in endowments.
- Endowments: \( \omega_h > 0 \).
Taxes Denominated in Commodities

• Taxes are denominated in terms commodity with real wealth taxes: \( \eta_h \in [0, \omega_h] \).

• The total taxes collected, \( \sum_h \eta_h \), are redistributed to the consumers.

• There is an “iceberg” costs of doing so. Thus:

\[
(1 - \delta) \sum_h \eta_h = \sum_h \nu_h, \quad \delta \in [0, 1].
\]

• After tax-transfer endowment is:

\[
\tilde{\omega}_h = \omega_h - \eta_h + \nu_h, \quad h = 1, 2, 3.
\]

• As there is a single commodity given post tax-transfer endowment, \( \tilde{\omega}_h \), there will be no trade.

• Hence, the post tax-transfer competitive equilibrium allocation is \( x_h = \tilde{\omega}_h \).
The Social Planner

- The planner has a utilitarian welfare function, with equal weights for the 3 consumers.

- The welfare function is:

  \[
  \max \sum_h \log(x_h) = \sum_h \log(\tilde{\omega}_h), \quad \tilde{\omega}_h = \omega_h - \eta_h + \nu_h.
  \]

- The planning problem is:

  \[
  \max_{\eta_h, \nu_h} \sum_h \log(\omega_h - \eta_h + \nu_h) \\
  \text{s.t.} \quad 0 \leq \eta_h \leq \omega_h, \\
  \quad \nu_h \geq 0 \\
  \quad \sum_h (1 - \delta)\eta_h = \sum_h \nu_h.
  \]
Optimal Lump-sum Commodity Taxes

Proposition 1

• When $\delta = 0$: Each individual receives the same post-taxation utility.

• When $\delta > 0$:
  • Each taxed individual receives the same post-tax utility.
  • Each subsidized individual receives the same post-tax utility.
  • A taxed individual receives larger post-tax utility than a subsidized individual.

• Subsidized individuals are inferior welfare machines. (We will see later under lump-sum money taxation that utilities will be the same within a group having the same information partition. Individuals with coarser information partitions are inferior welfare machines.)
With proportional taxes, $\gamma \in [0, 1]$, we have:

$$\tilde{\omega}_h = (1 - \gamma) \omega_h + \nu_h.$$

**Proposition 2**

*When $\delta = 0$ one can set $\gamma^* = 1$, and $\nu^*_h = \sum_h \omega_h / 3 = \bar{\omega}_h$ so that:*

$$\tilde{\omega}^*_h = (1 - \gamma^*) \omega_h + \bar{\omega}_h = \bar{\omega}_h.$$
Costly Proportional Commodity Taxes

• With costly taxes and transfers, the government resource constraint is: $\sum_h (1 - \delta) \gamma \omega_h = \sum_h \nu_h$.

• With equal transfers: $\nu_h = \sum_h (1 - \delta) \gamma \omega_h / 3 = (1 - \delta) \gamma \bar{\omega}$.

• There is only a single instruments, $\gamma$, to equalize three utilities.

• The planning problem is:

$$\max_{\gamma} \sum_h \log(x_h) = \sum_h \log(\tilde{\omega}_h) + \mu (1 - \gamma) + \lambda \gamma$$

with $\tilde{\omega}_h = (1 - \gamma) \omega_h + (1 - \delta) \gamma \bar{\omega}_h$
Proposition 3

- When $\delta = 0$, $\gamma^* = 1$. The utilities of consumers are equalized.
- When $\delta$ is small, $\gamma \in (0,1)$, and utilities are not equalized.
- When $\delta$ is large enough, $\gamma^* = 0$. 
Voting in the fully restricted economy with real taxation

- Now suppose that taxes are denominated in commodities so that: \( \tilde{\omega}_h = \omega_h - \gamma (\omega_h - \bar{\omega}) \), where \( \gamma \) is the uniform, real, "before-demo-grant" tax rate.

\[
\gamma^*_2 = \arg \max_{\gamma \in [0,1]} \{ u_2 [\omega_2 - \gamma (\omega_2 - \bar{\omega})] \} = \arg \max_{\gamma \in [0,1]} \{ u_2 [\omega_2 - \gamma (\omega_2 - \bar{\omega})] \} = 1;
\]

\[
\gamma^*_3 = \arg \max_{\gamma \in [0,1]} \{ u_3 [\omega_3 - \gamma (\omega_3 - \bar{\omega})] \} = \arg \max_{\gamma \in [0,1]} \{ u_3 [\omega_3 - \gamma (\omega_3 - \bar{\omega})] \} = 1;
\]

- Leveled utility with: \( x_h = \bar{\omega} \).
Voting in the fully restricted economy with costly commodity taxation at the rate $\gamma$

- **Iceberg Cost** (melting of chocolate, spoilage of apples, costly indexed security) of commodity taxation at rate $\delta \in [0, 1)$.

- Tax at rate $\gamma \in [0, 1]$ on real wealth $\omega_h$ with balanced-budget level real lump-sum demo-grant transfers $(1 - \delta)\gamma\bar{\omega}$.

- Tax-adjusted endowments are then:

  $$\tilde{\omega}_h = \omega_h - \gamma\omega_h + (1 - \delta)\gamma\bar{\omega} = \omega_h - \gamma[\omega_h - (1 - \delta)\bar{\omega}].$$

- Median voter will choose full-redistribution through commodity-taxation if spoilage cost $\delta$ is small enough. No volatility.
Taxes Denominated in Money

• 3 consumers (or 3 types): \( h = 1, 2, 3 \).

• Lump-Sum dollar taxation: \( \tau = (\tau_1, \tau_2, \tau_3) \in \mathbb{R}^3 \) with \( \tau \) balanced so that \( \tau_1 + \tau_2 + \tau_3 = 0 \). Hence, \( \tau = (\tau_1, \tau_2, -(\tau_1 + \tau_2)) \) is a 2-dimensional policy tool.

• Dollar tax based on income: \( \tau = (\tau_1, \tau_2, \tau_3) \in \mathbb{R}^3 \) but is restricted by

\[
\tau_h = \theta \omega_h - \theta \bar{\omega},
\]

where \( \theta \geq 0 \) is the tax rate, \( \omega_h \) is Mr. \( h \)'s ex-ante endowment of chocolate, and \( \bar{\omega} = (\omega_1 + \omega_2 + \omega_3) / 3 \) is the mean endowment. Hence \( \theta \) is a 1-dimensional policy tool. Notice that \( \tau_1 + \tau_2 + \tau_3 = 0 \).

• Bhattacharya, Guzman, & Shell (1998).
  • Fully anticipated exogenous lump-sum \( \tau \)-taxation denominated in dollars.
Balanced and Bonafide Taxes

- \( x_h = \omega_h - P^m \tau_h \).

- Summing over \( h \),
  \[
  \sum_h x_h = \sum_h \omega_h - P^m \sum_h \tau_h .
  \]

- Hence, Ricardo (in finite economy):
  - Either \( \tau \) is balanced or \( P^m = 0 \) or both
  - Balasko-Shell bonafide

...
Endogenous Money Taxes

• Fully Anticipated, Endogenous Money Taxation
  • $\tau$-Planner
  • $\theta$-Planner
  • $\theta$-Voting
  • Compare with costly commodity taxation.

• Goal
  • Analyze effects of Fully Anticipated Volatility on Equilibrium Taxes and Allocations
  • Real taxation is relatively inefficient in good times, but it does not create “excess” volatility.
  • Money taxation is relatively efficient, but does permit excess volatility. Money (and other paper assets) are the root of excess volatility.
• Hints that Price-Level Volatility might be bad:
  • Friedman
  • Shiller
  • Cass-Shell Immunity Theorem
• When Volatility is good:
  • Prescott-Townsend
  • Shell-Wright
• Focus of this talk: When Price-Level Volatility is not uniformly bad.
\textbf{Stochastic Economy}

- 2 states: $s = \alpha, \beta$.
  - $\pi(\alpha) + \pi(\beta) = 1$.
- $s$ can be interpreted as:
  - intrinsic uncertainty (random endowments, or monetary trembles), or
  - extrinsic uncertainty (“sunspots”).
- The commodity space $X_h = \mathbb{R}_{++}^2$.
- $\pi(\alpha) = \pi(\beta) = \frac{1}{2}$.
- Single commodity in each state and three consumers, $h = 1, 2, 3$.
- Consumers preferences identical: differ in endowments.
- $V_h(x_h(\alpha), x_h(\beta)) = \frac{1}{2} \log(x_h(\alpha)) + \frac{1}{2} \log(x_h(\beta))$, $h = 1, 2, 3$.
- $\omega_h(\alpha) = \omega_h(\beta) = \omega_h$. 
Time Line

- Expectations formed
- Taxes chosen
- Securities traded
- Taxes collected & securities paid
- Consumption
Restrictions

- Restriction #1: Incomplete Instruments: Tax rates have to be the same across states.

- Restriction #2, Asymmetric informations which implies:
  - Mr. 1 & Mr. 2 see the realization of $s$. They can trade on both the securities market and the spot market since $s$ is measurable with respect to their information. Their participation is unrestricted.
  - Mr. 3 cannot see the realization of $s$. He can only trade on the spot market. He cannot trade on the securities market.
    - His participation is restricted because of the information friction.
Monetary Taxes

- Ex-ante chocolate is the numeraire.
- \( p(s) \) is the price of chocolate in state \( s = \alpha, \beta \).
- \( p^m(s) \) is the price of money in state \( s = \alpha, \beta \).
- \( P^m(s) = p^m(s) / p(s) \) is the chocolate price of money (the inverse of the price level) in state \( s \).
- \( \tilde{\omega}_h(s) = \omega_h - P^m(s)\tau_h \) is the tax adjusted endowment in state \( s \).
- Base case: \( P^m(\alpha) = P^m(\beta) = P^m = 10 \).
- Mean-Preserving Spreads:
  - \( P^m(\alpha) = P^m - \sigma = 10 + \sigma \),
  - \( P^m(\beta) = P^m + \sigma = 10 + \sigma \).

where the standard deviation \( \sigma \) is non-negative.
There are two problems of multiplicity/indeterminacy.

1. Given the real (commodity) values of taxes and transfers, there may be multiple equilibria in the spot market.

2. There may be indeterminacy of the price level.

We focus on the effect of the second (price-level volatility) on the choice of taxes and transfers. Log-linear preferences, precluding the first multiplicity.
Consumer Problems

• Consumer 3 chooses $x_3(s) \in \mathbb{R}_{++}$ to

$$\text{maximize } \log(x_3(s))$$

subject to

$$p(s)x_3(s) = p(s)\omega_3 - p^m(s)\tau_3, \quad s = \alpha, \beta.$$  

• The budget constraints for Mr. 3 can be written as

$$x_3(s) = \tilde{\omega}_3(s), \quad s = \alpha, \beta.$$  

• Mr. 3 consumes his tax-adjusted endowment. (Formally, there are two Mr. 3’s: Mr. 3$\alpha$ and Mr. 3$\beta$.)
• Consumers $h = 1, 2$ choose $(x_h(\alpha), x_h(\beta)) \in \mathbb{R}^2_{++}$ to maximize

$$\frac{1}{2} \log(x_h(\alpha)) + \frac{1}{2} \log(x_h(\beta))$$

subject to

$$p(\alpha)x_h(\alpha) + p(\beta)x_h(\beta) = (p(\alpha) + p(\beta))\omega_h - (p^m(\alpha) + p^m(\beta))\tau_h.$$

so that $p(\alpha)x_h(\alpha) + p(\beta)x_h(\beta) = p(\alpha)\bar{\omega}_h(\alpha) + p(\beta)\bar{\omega}_h(\beta)$.

• Connection between extrinsic uncertainty & intrinsic uncertainty
  • Edgeworth box is now a proper rectangle, not a square.
Competitive Equilibrium

- A competitive equilibrium is a price vector 
  \((p(\alpha), p(\beta), p^m(\alpha), p^m(\beta))\) with \(p(s) > 0\) and \(p^m(s) \geq 0\) for 
  \(s = \alpha, \beta\) with demand equal supply, 
  \[
  \sum_{h=1,2,3} x_h(s) = \sum_{h=1,2,3} \omega_h, \text{ for } s = \alpha, \beta.
  \]

- Summing over the unrestricted consumers’ budget constraints yields 
  \[
  p(\alpha) \sum_{h=1,2} x_h(\alpha) + p(\beta) \sum_{h=1,2} x_h(\beta) = p(\alpha) \sum_{h=1,2} \tilde{\omega}_h(\alpha) + p(\beta) \sum_{h=1,2} \tilde{\omega}_h(\beta).
  \]
The equilibrium behavior of the unrestricted consumers can be described in terms of their state-specific tax-adjusted endowments in the tax-adjusted Edgeworth box of dimensions

\[ \sum_{h=1,2} \tilde{\omega}_h(\alpha) \times \sum_{h=1,2} \tilde{\omega}_h(\beta). \]

\[ [p^m(\alpha) + p^m(\beta)] \sum_{h=1,2,3} \tau_h = 0. \]

Set

\[ P^m(\alpha) = 10 - \sigma \text{ and } P^m(\beta) = 10 + \sigma. \]

The planner is Benthamite, i.e. he chooses \( \tau \) or \( \theta \) to maximize

\[ W = V_1 + V_2 + V_3. \]
The $\tau$-Planner

- Given $\tau_3$, $P^m(\alpha)$, and $P^m(\beta)$, Mr. 3 consumes his tax-adjusted endowments $\tilde{\omega}_3(\alpha)$ and $\tilde{\omega}_3(\beta)$.
- Mr. 1 & Mr. 2 trade in the tax-adjusted Edgeworth box. The optimal strategy for the $\tau$-planner is to equalize their marginal rates of substitution, i.e. to set $V_1 = V_2$ (for identical preferences).
  - The $\tau$-planner’s problem is in this case seemingly “1-dimensional”: choosing the lump-sum tax $\tau_3$ for the restricted consumer.
  - In general, the $\theta$-planner is less powerful than the $\tau$-planner: $\max W^\theta \leq \max W^\tau$. In our examples, the $\tau$-planner has 2 tools while the $\theta$-planner has only one tool. With more heterogeneity, the power of the $\tau$-planner relative to that of the $\theta$-planner is increased.
Proposition 4
\[ V_1 = V_2 \text{ under } \tau\text{-planning}. \]

Proposition 5
\[ \max_t W \geq \max_\theta W. \]

Proposition 6
*When the volatility \( \sigma = 0\), the \( \tau\)-planner levels, i.e. he gives everyone the same allocation. The \( \theta\)-planner also levels when volatility is zero.*
Proposition 7

When $\delta = 0$, and $\sigma = 0$, the optimal commodity tax plan and the optimal money tax plan yield the same allocations.
The welfare function is not necessarily concave in prices, so further theorems even in this simple world do not seem easy to come by.

- Mr. 1 & Mr. 2 are unrestricted.
- Mr. 3 is restricted.
- $P^m(\alpha) = 10 - \sigma$.
- $P^m(\beta) = 10 + \sigma$.
- $\pi(\alpha) = \pi(\beta) = 0.5$.
- $\tau$-Planner in blue.
- $\theta$-Planner in red.
Sample Calculation 1: Money Taxation

• Mr. 1 is rich, $\omega_1 = 80$.
• Mr. 2 is middle-class, $\omega_2 = 60$, and holds the median = the mean endowment.
• Mr. 3 is poor, $\omega_3 = 40$, and also disadvantaged in trading.
• $\bar{\omega} = (80 + 60 + 40)/3 = 60 = \omega_2 = \omega_{med}$.
• When $\sigma = 0$, under $\tau$-taxation $x_h(\alpha) = x_h(\beta) = \bar{\omega}$. Same for voting.
Vary $\text{std}(P^m)$ with mean = 10
with $\omega_1 = 80 \omega_2 = 60 \omega_3 = 40$; $\pi(\alpha) = 0.5$; $\lambda_1 = \lambda_2 = \lambda_3$
Vary std(Pm) with mean = 10
with $\omega_1 = 80$, $\omega_2 = 60$, $\omega_3 = 40$; $\pi(\alpha) = 0.5$; $\lambda_1 = \lambda_2 = \lambda_3$.
Sample Calculation 2: Money Taxation

- Mr. 1 is rich, $\omega_1 = 80$.
- Mr. 2 and Mr. 3 have the same endowment, $\omega_2 = \omega_3 = 40$, but Mr. 3 is disadvantaged in trading.
- $\bar{\omega} = 160/3$, $\omega_{med} = 40$.
- When $\sigma = 0$, $u_1 = u_2 = u_3$ under $\tau$-taxation.
- Mr. 3 is very “expensive” for the $\tau$-planner: For $\sigma > 0$, $u_2 > u_3$. Furthermore, $u_3$ blue is below $u_3$ red. The planner does not give Mr. 3 more because of his handicap. He gives him less, especially when he can target Mr. 3 by name. Mr. 3 is an inferior welfare machine.
Vary std($P_m$) with mean = 10
with $\omega_1 = 80$, $\omega_2 = 40$, $\omega_3 = 40$; $\pi(\alpha) = 0.5$; $\lambda_1 = \lambda_2 = \lambda_3$
Vary \( \text{std}(P_m) \) with mean = 10
with \( \omega_1 = 80 \), \( \omega_2 = 40 \), \( \omega_3 = 40 \); \( \pi(\alpha) = 0.5; \lambda_1 = \lambda_2 = \lambda_3 \)
Polar cases: $\tau$-Planner

- If everyone is unrestricted, then the equilibrium allocations are state symmetric (Cass-Shell Immunity Theorem).
- If everyone is restricted, then only when price expectations are state symmetric, i.e. $P^m(\alpha) = P^m(\beta)$, are equilibrium allocations state symmetric.
- This is different from commodity taxes where allocations are necessarily state symmetric.
- The intuition for this is that the planner has incomplete instruments and tries to compensate for potentially differing real taxes and transfers in the two states, but is unable to do so.
Majority voting in the fully restricted money-taxation economy

- Assume $\omega_1 > \bar{\omega} > \omega_2 > \omega_3$. Mean wealth > median wealth.
- $P^m(\alpha) > P^m(\beta) > 0$.
- Preferred money tax rates $\theta$:

$$\theta^*_2 = \arg \max_{\theta \in [0, \frac{1}{P^m(\alpha)}]} \left\{ \begin{array}{c} \pi(\alpha)u_2 [\omega_2 - \theta P^m(\alpha)(\omega_2 - \bar{\omega})] + \\
\pi(\beta)u_2 [\omega_2 - \theta P^m(\beta)(\omega_2 - \bar{\omega})] \end{array} \right\} = \frac{1}{P^m(\alpha)}$$

$$\theta^*_3 = \arg \max_{\theta \in [0, \frac{1}{P^m(\alpha)}]} \left\{ \begin{array}{c} \pi(\alpha)u_3 [\omega_3 - \theta P^m(\alpha)(\omega_3 - \bar{\omega})] + \\
\pi(\beta)u_3 [\omega_3 - \theta P^m(\beta)(\omega_3 - \bar{\omega})] \end{array} \right\} = \frac{1}{P^m(\alpha)}$$
Voting in the fully restricted money-taxation economy

The marginal expected utility of the tax rate is positive for Mr. 2 and Mr. 3:

\[
(\bar{\omega} - \omega_h) \sum_{s=\alpha}^{\beta} P^m(s) \pi(s) u'_h [\omega_h - \theta P^m(s)(\omega_h - \bar{\omega})] > 0,
\]

Equilibrium indirect utility is:

\[
\pi(\alpha)u_h(\bar{\omega}) + \pi(\beta)u_h \left[ \omega_h - \frac{P^m(\beta)}{P^m(\alpha)}(\omega_h - \bar{\omega}) \right].
\]

Generically, \( \bar{\omega} \neq \left[ \omega_h - \frac{P^m(\beta)}{P^m(\alpha)}(\omega_h - \bar{\omega}) \right] \).

Sunspots matter. The SSE allocation is not a mere randomization over CE except in trivial cases. Contrast this with Cass and Shell (1983).
Comparing $\theta$-money taxation vs. $\gamma$-commodity taxation

- If $\delta > \min\left(\frac{\bar{\omega} - \omega_h}{\bar{\omega}}, \bar{\delta}\right)$, money taxation is preferred., by Mr. h.
- If $\delta < \min\left(\frac{\bar{\omega} - \omega_h}{\bar{\omega}}, \bar{\delta}\right)$, commodity taxation is preferred., by Mr. h.

- The critical spoilage rate $\bar{\delta}$ is increasing in $(P^m(\alpha) / P^m(\beta))$. A low inflation rate is favorable for the adoption of money-taxation.

- The critical spoilage rate $\bar{\delta}$ is decreasing in $\pi(\alpha)$. A lower probability of inflation favorable for the adoption of money-taxation.