$$\begin{aligned} Y &= C(Y-T)+Z+G \\ dY &= C'(Y-T)dY+dG \\ (1-MPC)dY &= dG \end{aligned}$$

$$\text{GEM: } \frac{dY}{dG} = \frac{\mathbf{1}}{\mathbf{MPS}}$$

$$\text{MPS} < \frac{1}{20}$$

Theorectical GEM > 20

way too big

$$
\begin{aligned}
dY &= C'(Y-T)dY - C'dT \\
(1-\text{MPC})dY &= -MPCdT \\
\frac{dY}{dT} &= -\frac{\text{MPS}}{\text{MPC}} = -\frac{\text{MPS}}{1-\text{MPS}}
\end{aligned}
$$

TCM: $\frac{dY}{dT} = \frac{-1}{MPS-1} = -\{\textbf{GEM} - \textbf{1}\}$

BB $dG = dT$

$$\begin{aligned} dY &= C'(Y - T)\, dY - C' dT + dG \\ (1 - MPC)\, dY &= -MPC dG + dG \end{aligned}$$

BBM: $\left(\dfrac{dY}{dG}\right)_{dT=dG} = 1$

$$p_Y Y = p_C C + p_Z Z + p_G G$$

Standard IS or IS-LM assumes

$$p_Y = p_C = p_Z = p_G \ (= 1) .$$

This can be explained by "flat" production as in the one-sector model of production:

$$Y = F(K, L),$$

where F is increasing and concave with

$$F_K = \frac{\partial F}{\partial K} > 0$$

$$F_L = \frac{\partial F}{\partial L} > 0$$

$$F_{KK} < 0$$

$$F_{LL} < 0$$

$$
\begin{aligned}
2Y &= F(2K, 2L) \\
\theta Y &= F(\theta K, \theta L) \text{ for } \theta > 0
\end{aligned}
$$

CRS. F is homogeneous of degree one. (F is linearly homogeneous.)

Profit Maximization:

$$
\begin{aligned}
\pi &= p_Y Y - wL - rK \quad \text{set } p_Y = 1 \\
&= Y - wL - rK \\
&= F(K, L) - wL - rK \\
\frac{\partial \pi}{\partial K} &= F_K - r = 0 \\
\frac{\partial \pi}{\partial L} &= F_L - w = 0 \\
\frac{\partial \pi}{\partial K} &= F_K - r = 0
\end{aligned}
$$

$$
\begin{aligned}
MPP_K &= r \\
MPP_L &= w
\end{aligned}
$$

CRS allows us to focus on scale-free economy:

$$\begin{aligned} y &= Y/L \\ c &= C/L \\ z &= Z/L \\ k &= K/L \end{aligned}$$

$$\begin{aligned} \frac{Y}{L} &= F\left(\frac{K}{L}, \frac{L}{L}\right) = F(k, 1) \\ y &= f(k) \end{aligned}$$

Example: Log-linear or Cobb-Douglas

- Paul Douglas
- Charles Cobb

$$
\begin{aligned}
Y &= AK^{\alpha}L^{1-\alpha} \\
\frac{\partial Y}{\partial K} &= \alpha AK^{\alpha-1}L^{1-\alpha} \\
K\frac{\partial Y}{\partial K} &= \alpha AK^{\alpha}L^{1-\alpha} = \alpha Y = rK \\
rK/Y &= \alpha!\ wL/Y = (1-\alpha)! \\
\frac{Y}{L} &= \frac{AK^{\alpha}}{L^{\alpha}} \\
y &= Ak^{\alpha}
\end{aligned}
$$

- Factor shares constant with CD and MP factor reward
- To show: If factors share are constant, then CD PF.

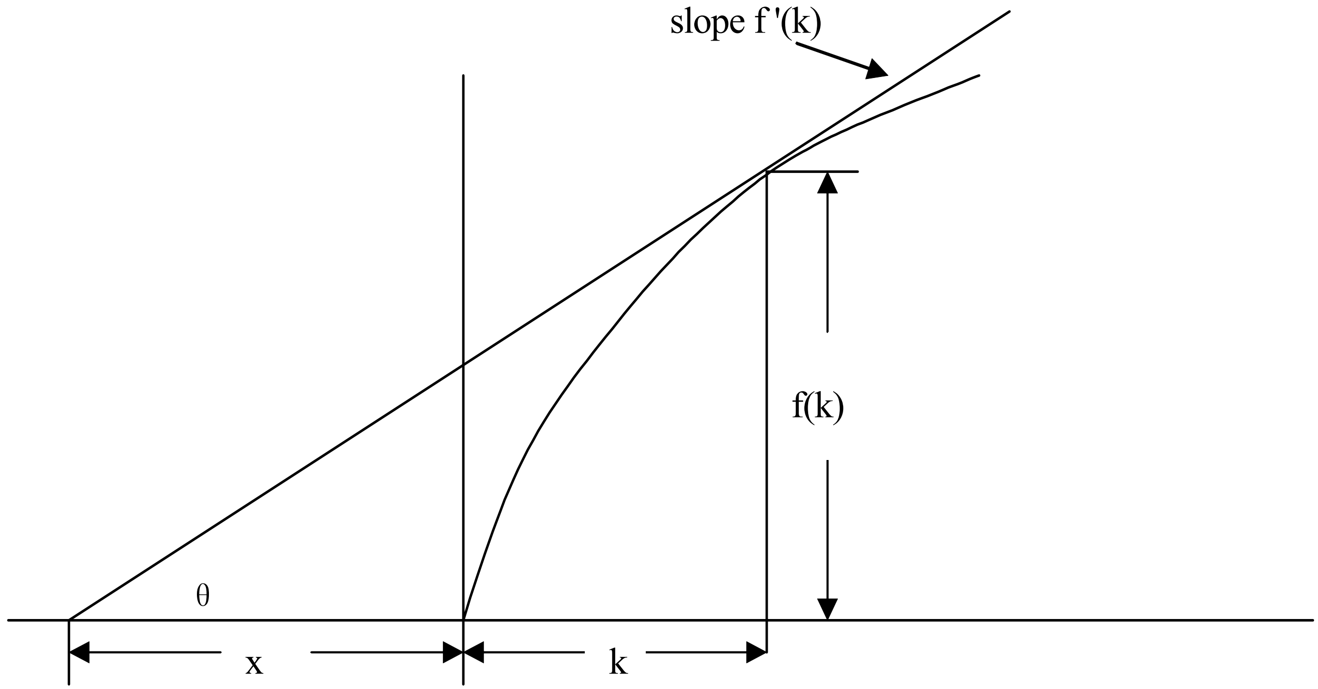
General 1-sector

$$\begin{aligned} Y &= Lf\left(\frac{K}{L}\right) \\ MPP_K &= \frac{Lf'\left(\frac{K}{L}\right)}{L} = f'(k) = r \\ MPP_L &= f\left(\frac{K}{L}\right) - Lf'\left(\frac{K}{L}\right)\left(\frac{K}{L^2}\right) \\ &= f(k) - kf'(k) = w \end{aligned}$$

Euler adding-up theorem:

$$\begin{aligned} & rK + wL \\ = & f'(k)\,kL + \Big(f(k) - kf'(k)\Big)L \\ = & Lf(k) = Y \end{aligned}$$

- CRS allows for competition with zero profits
- IRS overexhausts output
- DRS leaves profits


slope f '(k)
f(k)
θ
x
k

$$\tan\theta = \frac{f(k)}{k+x} = f'(k)$$

$$x = \frac{f - kf'}{f'} = \frac{w}{r} = \omega$$

$$k(\omega) : \frac{dk(\omega)}{d\omega} > 0$$

$$\omega(k) : \frac{d\omega(k)}{dk} > 0$$

One to one relationship between factor-price (or wage-rentals) ratio and captial intensity (or capital-labor ratio).

$$\omega = \frac{f(k) - kf'(k)}{f'(k)}$$

$$\omega = \frac{f(k)}{f'(k)} - k$$

$$\frac{d\omega}{dk} = -1 + \frac{[f'(k)]^2 - f(k)f''(k)}{[f'(k)]^2}$$

$$\boxed{\frac{d\omega}{dk} = \frac{-f(k)f''(k)}{[f'(k)]^2} > 0}$$

$$\boxed{\frac{dk}{d\omega} = \frac{-[f'(k)]^2}{f(k)\,f''(k)} > 0}$$