Economics 613: Macroeconomics I
Fall 2006
Cornell University
Problem Set #10
Due: Monday, November 13, 2006

1 Overlapping Generations

Consider an overlapping generations models in which the agents have 2-period lives, and there is one commodity per period, i.e. $\ell = 1$.

Assume stationary endowments:

$$\begin{aligned} \omega_0^1 &= B > 0 \text{ for } t = 0 \\ (\omega_t^t, \omega_t^{t+1}) &= (A, B) \gg 0 \text{ for } t = 1, 2, \ldots, \end{aligned}$$

stationary preferences:

$$\begin{aligned} u_0(x_0^1) &= D \ln x_0^1 \text{ for } t = 0 \\ u_t(x_t^t, x_t^{t+1}) &= C \ln x_t^t + D \ln x_t^{t+1} \text{ for } t = 1, 2, \ldots, \end{aligned}$$

and passive fiscal policy:

$$m_0^1 = 2, \qquad m_t^s = 0 \text{ otherwise,}$$

where the goods price of money is $p^m \geq 0$.

Precisely plot (use graph paper if necessary) the offer curve in excess demand space ($x_t^t - \omega_t^t$, $x_t^{t+1} - \omega_t^{t+1}$) for Mr. $t \geq 1$. Plot the reflected offer curve, and analyze the global dynamics for each of the following cases:

(a) $A = 2, B = 4, C = 4, D = 2$
(b) $A = 2, B = 6, C = 1, D = 3$
(c) $A = 10, B = 5, C = 4, D = 6$
(d) $A = 2, B = 2, C = 2, D = 5$

Is there a pattern?

Derive the conditions for a "Samuelson" versus a "Classical" economy and relate them to the above.

Let $m_0^1 = -1$ (negative money). Redo all the exercises above. Is there a pattern? What happens to the Samuelson economy when going from positive money to negative money? The classical economy? [Hint: Be sure to plot the **FULL** reflected offer curve.]

2 Overlapping Generations Under Transfers

Assume the following setup in an overlapping generations model:

$$\begin{aligned} u_0(c_0^1) &= c_0^1 \text{ for } t = 0, \\ u_t(c_t^t, c_t^{t+1}) &= c_t^t + c_t^{t+1} \text{ for } t = 1, 2, \ldots, \end{aligned}$$

where each agent t is endowed with one chocolate in each period (i.e. one chocolate in period t and one chocolate in period $t+1$), and Mr. 0 has one unit of chocolate in period 1, i.e.

$$\begin{aligned} \omega_0^1 &= 1 \text{ for } t = 0 \\ (\omega_t^t, \omega_t^{t+1}) &= (1, 1) \text{ for } t = 1, 2, \ldots. \end{aligned}$$

Find a system of transfers from young to old that strictly increases every person's utility from his autarky level [Hint: look for an infinite series that sums to a positive number, which is less than or equal to unity]. Show that if a net total of one chocolate is brought forward through this infinite process, then the resulting allocation is Pareto optimal.