1 Overlapping Generations

Consider an overlapping generations model in which the agents have 2-period lives, and there is one commodity per period, i.e. \( t = 1 \).

Assume stationary endowments:

\[
\omega^t_0 = \begin{cases} 
B > 0 & \text{for } t = 0 \\
(A, B) & \text{for } t = 1, 2, \ldots
\end{cases}
\]

stationary preferences:

\[
u^0_0(x^0_0) = D \ln x^0_0 \text{ for } t = 0
\]

\[
u^t(x^t_t, x^{t+1}_t) = C \ln x^t_t + D \ln x^{t+1}_t \text{ for } t = 1, 2, \ldots,
\]

and passive fiscal policy:

\[
m^1_0 = 2, \quad m^s_t = 0 \text{ otherwise},
\]

where the goods price of money is \( p^m \geq 0 \).

Precisely plot (use graph paper if necessary) the offer curve in excess demand space \((x^t_t - \omega^t_t, x^{t+1}_t - \omega^{t+1}_t)\) for Mr. \( t \geq 1 \). Plot the reflected offer curve, and analyze the global dynamics for each of the following cases:

(a) \( A = 2, B = 4, C = 4, D = 2 \)
(b) \( A = 2, B = 6, C = 1, D = 3 \)
(c) \( A = 10, B = 5, C = 4, D = 6 \)
(d) \( A = 2, B = 2, C = 2, D = 5 \)

Is there a pattern?

Derive the conditions for a "Samuelson" versus a "Classical" economy and relate them to the above.

Let \( m^1_0 = -1 \) (negative money). Redo all the exercises above. Is there a pattern? What happens to the Samuelson economy when going from positive money to negative money? The classical economy? [Hint: Be sure to plot the FULL reflected offer curve.]

2 Overlapping Generations Under Transfers

Assume the following setup in an overlapping generations model:

\[
u^0_0(c^0_0) = c^0_0 \text{ for } t = 0,
\]

\[
u^t(c^t_t, c^{t+1}_t) = c^t_t + c^{t+1}_t \text{ for } t = 1, 2, \ldots
\]

where each agent \( t \) is endowed with one chocolate in each period (i.e. one chocolate in period \( t \) and one chocolate in period \( t + 1 \)), and Mr. \( 0 \) has one unit of chocolate in period 1, i.e.

\[
\omega^t_0 = 1 \text{ for } t = 0
\]

\[
(\omega^t_t, \omega^{t+1}_t) = (1, 1) \text{ for } t = 1, 2, \ldots
\]

Find a system of transfers from young to old that strictly increases every person’s utility from his autarky level [Hint: look for an infinite series that sums to a positive number, which is less than or equal to unity]. Show that if a net total of one chocolate is brought forward through this infinite process, then the resulting allocation is Pareto optimal.