1 Inventive Activity and Capital Accumulation

\[ Y = A^\alpha K^\beta L^{1-\beta} \]
\[ Y = C + Z + R \]
\[ \dot{K} = Z - \mu K \]
\[ \dot{A} = R - \rho A \]
\[ R = \tau Y \]
\[ Z = (1 - \tau) sY, \]

where \( \alpha > 0, \ 0 < \beta < 1, \ 0 < \tau < 1, \ 0 < s < 1, \ \rho > 0, \ \mu > 0. \)

Draw and analyze the complete phase diagrams for:

(a) \( \alpha = 1 \quad \beta = 1 \]
\[ \tau = 1/5 \quad s = 1/2 \]
\[ \rho = 1/8 \quad \mu = 1/10 \]

(b) \( \alpha = 3/4 \quad \beta = 1/3 \]
\[ \tau = 1/4 \quad s = 1/2 \]
\[ \rho = 1/8 \quad \mu = 1/10 \]

(c) \( \alpha = 2/3 \quad \beta = 1/5 \]
\[ \tau = 2/5 \quad s = 2/3 \]
\[ \rho = 1/8 \quad \mu = 1/10 \]

(d) \( \alpha = 2/3 \quad \beta = 1/3 \]
\[ \tau = 2/5 \quad s = 2/3 \]
\[ \rho = 1/8 \quad \mu = 1/10 \]

"Draw and analyze" means fully describing the dynamical system, including as much as possible:
(i) the phase diagram, (ii) the associated linear system, (iii) the economic implications, (iv) the
stability of the system, and (v) the robustness of the assumptions.

Note: In parts (a) and (b), the production function exhibits increasing returns in the reproducible factors (\( 1 + 1 > 1, \ 3/4 + 1/3 > 1 \)), in part (c) the production function exhibits decreasing returns to the reproducible factors (\( 2/3 + 1/5 < 1 \)), and in part (d) it exhibits constant returns (\( 2/3 + 1/3 = 1 \)).

2 Externalities

Assume that the production function for firm \( i \) is

\[ Y_i = F(K_i, L_i, K) = 10(K_i)^{1/3}(L_i)^{2/3}(K)^{1/8}, \]

where \( K_i \) is capital employed in firm \( i \), and \( K \) is the aggregate capital stock (namely, \( K = \sum_i K_i \)).

Firm \( i \) is so small that its effect on \( K \) is negligible, and firm \( i \) believes that all firms act on the same belief. Assume that \( F \) is homogeneous of degree one in \( K_i \) and \( L_i \). Set up the RCK model for this situation. Solve it. Analyze it.