

Economics 613: Macroeconomics I

Fall 2006

Cornell University

Problem Set #13

Due: 4 pm, Thursday, December 7, 2006

1 Inventive Activity and Capital Accumulation

$$\begin{aligned}Y &= A^\alpha K^\beta L^{1-\beta} \\Y &= C + Z + R \\ \dot{K} &= Z - \mu K \\ \dot{A} &= R - \rho A \\ R &= \tau Y \\ Z &= (1 - \tau) s Y,\end{aligned}$$

where $\alpha > 0$, $0 < \beta < 1$, $0 < \tau < 1$, $0 < s < 1$, $\rho > 0$, $\mu > 0$.

Draw and analyze the complete phase diagrams for:

(a)

$$\begin{aligned}\alpha &= 1 & \beta &= 1 \\ \tau &= 1/5 & s &= 1/2 \\ \rho &= 1/8 & \mu &= 1/10\end{aligned}$$

(b)

$$\begin{aligned}\alpha &= 3/4 & \beta &= 1/3 \\ \tau &= 1/4 & s &= 1/2 \\ \rho &= 1/8 & \mu &= 1/10\end{aligned}$$

(c)

$$\begin{aligned}\alpha &= 2/3 & \beta &= 1/5 \\ \tau &= 2/5 & s &= 2/3 \\ \rho &= 1/8 & \mu &= 1/10\end{aligned}$$

(d)

$$\begin{aligned}\alpha &= 2/3 & \beta &= 1/3 \\ \tau &= 2/5 & s &= 2/3 \\ \rho &= 1/8 & \mu &= 1/10\end{aligned}$$

"Draw and analyze" means fully describing the dynamical system, including as much as possible: (i) the phase diagram, (ii) the associated linear system, (iii) the economic implications, (iv) the stability of the system, and (v) the robustness of the assumptions.

Note: In parts (a) and (b), the production function exhibits increasing returns in the reproducible factors ($1 + 1 > 1$, $3/4 + 1/3 > 1$), in part (c) the production function exhibits decreasing returns to the reproducible factors ($2/3 + 1/5 < 1$), and in part (d) it exhibits constant returns ($2/3 + 1/3 = 1$).

2 Externalities

Assume that the production function for firm i is

$$Y_i = F(K_i, L_i, K) = 10(K_i)^{1/3}(L_i)^{2/3}(K)^{1/8},$$

where K_i is capital employed in firm i , and K is the aggregate capital stock (namely, $K = \sum_i K_i$).

Firm i is so small that its effect on K is negligible, and firm i believes that all firms act on the same belief. Assume that F is homogeneous of degree one in K_i and L_i . Set up the RCK model for this situation. Solve it. Analyze it.