1 One-sector Production

\[ Y = F(K, L) \]
\[ C + Z = Y \]
\[ K > 0, \ L > 0, \ C \geq 0, \ Z \geq 0 \]

(a) Given \( K \) and \( L \), what is the PPF (production possibility frontier)? Be precise.

Let \( F(K, L) = 2K^{\frac{4}{3}}L^{\frac{2}{3}} \). Assume perfect competition.

(b) What do we know about the competitive shares (of the output) for capital and labor?

(c) How is the composition of output affected by output prices?

(d) If \((K, L) = (8, 8)\) what are the rental rate on capital \((r)\) and the wage rate for labor \((w)\)?

These are the input (or factor) prices.

The ratio \( \omega = \frac{w}{r} \) is the factor price ratio; let \( k = \frac{K}{L} \).

(e) How does \( \omega \) vary with \( k \)?

2 More One-sector Model

(a) Show that if the production function is

\[ AK^aL^{1-a}, \]

where \( 0 < a < 1 \), that capital's share is \( a \) and labor's share is \( 1 - a \).

(b) Assume constant returns to scale and perfect competition. Show that if capital’s share is \( a \) and labor’s share is \( 1 - a \), then the production function is

\[ AK^aL^{1-a}, \]

where \( A > 0 \) is an undetermined constant.