Economics 613: Macroeconomics I
Fall 2006
Cornell University
Problem Set #8
Due: Friday, Oct. 27, 2006

1 Phase Diagrams

Draw the phase diagrams for the following and do the full dynamic analysis:

(a)

$$\begin{aligned} Y &= AK, \\ \dot{K} &= s_K Y - \mu K, \\ \dot{A} &= s_A Y - \rho A, \end{aligned}$$

where $s_K > 0,\ s_A > 0,\ s_K + s_A < 1,\ A(0) > 0,\ K(0) > 0,\ \mu, \rho > 0.$

(b) From part (a), replace AK with $A^{\alpha}K^{\beta}$, where $\alpha > 0$, $\beta > 0$, and $\alpha + \beta < 1$.

(c) From part (b), now consider the case $\alpha + \beta > 1$.

2 Calculus of Variations

(i) *Finite time without discounting.* Solve for the optimal value of $x(t)$ which solves

$$\min \Phi = \int_a^b \sqrt{1 + (\dot{x})^2} dt$$

$$\begin{aligned} s.t.\ x(a) &= \alpha \\ x(b) &= \beta. \end{aligned}$$

where x is a state variable, and $\alpha, \beta, a, b > 0$ are scalars. Graph $x(t)$ as a function of time.

(ii) *Finite time with discounting.*

$$\max \Phi = \int_0^T \{D[x(t), \dot{x}(t)]x(t) - C\,[D[x(t), \dot{x}(t)]]\}\, e^{-\delta t} dt$$

$$\begin{aligned} s.t.\ x(0) &= x_0 \\ x(T) &= x_T, \end{aligned}$$

where δ is a scalar. *Either* assume $\delta > 0$ and solve for the second-order differential equation, or assume $\delta = 0$ and solve for the first-order differential equation.

(iii) *Infinite time with discounting.* Obtain the system of differential equations in $\dot{k}$ and $\dot{q}$ resulting from

$$\max \Phi = \int_0^\infty U(c(t)) e^{-\delta t} dt$$

$$\begin{aligned} s.t.\ \dot{K}(t) &= Z(t) - \mu K(t) \\ F(K(t), L(t)) &= AK(t)^{\alpha} L(t)^{1-\alpha} = C(t) + Z(t) \\ \dot{L}(t)/L(t) &= n \\ U(x) &= \ln x \\ and\ K(0) &= K_0, \end{aligned}$$

where $k(t) = K(t)/L(t)$ is our state variable, $c(t)$ is our control variable, and $\alpha,\ A,\ \delta,$ and $\lambda = n + \mu$ are scalar parameters, where $0 < \alpha < 1$, $A, \delta, \lambda > 0$.