1 Solow Growth Model, Continuous Time

1. Impulse diagrams. Suppose that the economy is in a steady-state, but at time $t_0$ the savings rate jumps from $s_b$ to $s_a$ where $s_a < s_b$ (as given in the diagram). Complete the graphs on the following page.

2. Describe how the following affects the continuous-time Solow diagram and the steady state.
   (a) The depreciation rate falls.
   (b) The production function is Cobb-Douglas, and capital’s share rises.
   (c) Workers exert more effort, so it takes 50 minutes to do what it formerly took 60 minutes to do.

2 Leontief Production Function

$$C + Z = Y = \min\{aK, bL\}$$

(a) Write down the intensive production function and plot $y$ on the vertical axis versus $k$ on the horizontal axis.

(b) Using this production function, study the full dynamics of (i) the Solow model in continuous time, (ii) the Ramsey-Cass-Koopmans model in continuous time.

3 Optimal Growth

For the following optimal growth problem

$$\max \int_0^\infty U(c(t))e^{-\delta t}dt$$

s.t. $c(t) = f(k(t)) - \lambda k(t) - \dot{k}(t)$

$k(0) = k_0, \delta > 0$

$U'(c) > 0, U''(c) < 0, \text{ for } c > 0$

$U'(0) = +\infty, f(k) > 0, f''(k) > 0, f''(k) < 0 \text{ for } k > 0,$

show that the path satisfying the Euler equation (or the Hamiltonian equations) and tending to the rest point $(q^*, k^*)$ is the optimal path.

4 Optimal Growth, Cont.’d.

For problem 3, show that the Euler equations are equivalent to (or closely related to) the corresponding Hamiltonian equations from Pontryagin’s Maximum Principle.

5 Ramsey-Cass-Koopmans Model

Suppose the economy is at balanced growth, but that at time $t_0$ a tax rate $\tau$ on capital income is unexpectedly instituted. Assume that the proceeds of the tax are distributed to people as lump-sum transfers.

(a) What is the after-tax rate of return to the household?
(b) How does the tax rate affect the $\dot{q} = 0$ locus? The $\dot{k} = 0$ locus?
(c) How does the new balanced growth path compare to the old?
Diagrams in Problem Set #9

\[ \text{s} \]

\[ \text{s}_b \]

\[ \text{s}_a \]

\[ t_0 \]

\[ \text{t} \]

\[ \text{k-dot} \]

\[ t_0 \]