Lecture Notes #11


Separate sources of sunspot equilibria:

- restricted market participation (OG and other restrictions)
- OG double infinity (even without restricted participation)
- incomplete markets
- imperfect competition
- indivisible goods and other non-convexities
- search
- asymmetric information
- externalities

See Karl Shell on SSE: karlshell.com/sum1.html

\[ l = 1, \ h \in H, \ X_h = R_{++}, \ p^m \geq 0 \]

\[
\max u_h(x_h) \\
\text{s.t. } x_h = \omega_h - p^m \tau_h \\
\sum_H x_h = \sum_H \omega_h \\
\sum_H \tau_h = 0
\]
Parameters:

\[ H = \{1, 2, 3\} \]

\[ \omega = (\omega_1, \omega_2, \omega_3) = (20, 10, 5) \]

\[ \tau = (\tau_1, \tau_2, \tau_3) = (5, 0, -5) \]

CE allocations:

\[ \{(x_1, x_2, x_3) \in \mathbb{R}^3_+ | x_1 = 20 - 5p^m, x_2 = 10, x_1 = 5 + 5p^m, p^m \in \mathbb{R}^3_+ \} \]

\[ p^m \in [0, 4) \]

\[ X_{CE}^m = \{(x_1, x_2, x_3) | x_1 = 20 - 5p^m, x_2 = 10, x_1 = 5 + 5p^m, p^m \in [0, 4)\} \]
Corresponding sunspots economy with \( s = \alpha, \beta \):

Endowments: \( \omega_h(\alpha) = \omega_h(\alpha) = \omega_h \)

Taxes: \( \tau_h(\alpha) = \tau_h(\alpha) = \tau_h \)

Preferences: \( V(x_h(\alpha), x_h(\beta)) = \pi(\alpha)u_h(x_h(\alpha)) + \pi(\beta)u_h(x_h(\beta)) \)

\( H = G^0 \cup G^1 \)

\( G^0 \) can trade securities. \( G^1 \) cannot
$h \in G^1$ choose $x_h(s) \in R_{++}$ to

$$\max u_h(x_h(s))$$

s.t. $p(s)x_h(s) = p(s)\omega_h - p^m(s)\tau_h$ \hspace{1em} for $s = \alpha, \beta$

Like two separate guys Mr $h\alpha$ and Mr $h\beta$

$$\tilde{\omega}_h(s) = \omega_h - \left(\frac{p^m(s)}{p(s)}\right)\tau_h$$

$$p(s)x_h(s) = p(s)\tilde{\omega}_h(s)$$

$$\sum_{G^1} x_h(s) = \sum_{G^1} \tilde{\omega}_h(s) \hspace{1em} \text{for } s = \alpha, \beta$$
Timeline for the OG interpretation of the restrictions on market participation.
\( h \in G^0 \) choose \((x_h(\alpha), x_h(\beta)) \in R^2_{++}\) to

\[
\max \pi(\alpha)u_h(x_h(\alpha)) + \pi(\beta)u_h(x_h(\beta))
\]

\(\text{s.t. } p(\alpha)x_h(\alpha) + p(\beta)x_h(\beta) = (p(\alpha) + p(\beta))\omega_h - (p^m(\alpha) + p^m(\beta))\tau_h \)

or

\( p(\alpha)x_h(\alpha) + p(\beta)x_h(\beta) = p(\alpha)\tilde{\omega}_h(\alpha) + p(\beta)\tilde{\omega}_h(\beta) \)

\[
\sum_H x_h(s) = \sum_H \omega_h
\]
Parameters:

\[ u_h(x_h(s)) = \log x_h(s), \quad \pi = (\pi(\alpha), \pi(\beta)) = (3/4, 1/4) \]

Partial restricted participation

\[ G^0 = \{1.2\} \quad G^1 = \{3\} \]

\[ x_3(\alpha) = \tilde{\omega}_3(\alpha) \quad x_3(\beta) = \tilde{\omega}_3(\beta) \]

\[ x_h(s) = \frac{\pi(s)}{p(s)} w_h \quad s = \alpha, \beta, \quad h = 1, 2 \]

\[ w_h = (p(\alpha) + p(\beta))\omega_h - (p^m(\alpha) + p^m(\beta))\tau_h \quad h = 1, 2 \]

\[ x_1(s) + x_2(s) = \tilde{\omega}_1(s) + \tilde{\omega}_2(s) \quad s = \alpha, \beta \]
Example 1:

\[ p^m(\alpha) = 1 \quad p^m(\beta) = 2 \]

\[ (\tilde{\omega}_1(\alpha), \tilde{\omega}_1(\beta)) = (15, 10) \]

\[ (\tilde{\omega}_1(\alpha), \tilde{\omega}_1(\beta)) = (10, 10) \]

\[ (\tilde{\omega}_1(\alpha), \tilde{\omega}_1(\beta)) = (10, 15) = (x_3(\alpha), x_3(\beta)) \]
Example 2: High-Volatility

\[ p^m(\alpha) = 1 \quad p^m(\beta) = 5 \]

\[(\tilde{\omega}_1(\alpha), \tilde{\omega}_1(\beta)) = (15, -5)\]

\[(\tilde{\omega}_1(\alpha), \tilde{\omega}_1(\beta)) = (10, 10)\]

\[(\tilde{\omega}_1(\alpha), \tilde{\omega}_1(\beta)) = (10, 30) = (x_3(\alpha), x_3(\beta))\]
Equilibrium:

\[ p(\alpha) / p(\beta) = \frac{3}{5} \]

\[(x_1(\alpha), x_1(\beta)) = (5, 1)\]

\[(x_2(\alpha), x_2(\beta)) = (20, 4) \neq (10, 10)\]

\[(x_3(\alpha), x_3(\beta)) = (10, 30)\]

The scare resource for Mr. 1 and Mr. 2 is the good in \(\beta\)

Mr. 1 owns -5 units, while Mr. 2 owns +10 units, so Mr. 2 has a much better equilibrium allocation.