Lecture Notes #14

Review of Bank Runs
$$U_I(C_i^1, C_i^2) = \begin{cases} 
\bar{u} + u(C_i^1 + C_i^2 - 1) & \text{if } C_i^1 \geq 1 \\
\beta \bar{u} + u(C_i^1 + C_i^2 - 1) & \text{if } C_i^1 < 1 
\end{cases}$$

(1)

and

$$U_P(C_P^1, C_P^2) = \bar{u} + u(C_P^1 + C_P^2 - 1),$$

Here: $I$ and $P$ denote actually impatient and actually patient. Assuming that it is always best to take a consumption opportunity if one can above equations give utility as a function of withdrawals. Feasibility is not relevant here. Assume that left-over cash balance (in or out of bank) is the argument of $u(\cdot)$.(Alternatively: at the end of period 2, left-over cash is deposited)
continuum of **ex-ante** identical consumers

$y$ units of consumption

fraction $\alpha$ are impatient

$\alpha$ is uncertain, $\alpha_1$ is actually withdrawal

$\bar{u}$ for best consumption opportunity

$\beta \bar{u}$ for next-best $\beta < 1$

utility of "left-over" bank balance, $u(\cdot)$

$f(\alpha)$ defined on $[0, \bar{\alpha}]$, where $0 < \bar{\alpha} < 1$, e.g. $\bar{\alpha} = 1/2$
\[ f_P(\alpha) = \frac{(1 - \alpha)f(\alpha)}{\int_0^\alpha (1 - a)f(a)da} \cdot \]

Important for ICC.

Bayes Rule:

\[
\text{Prob}(A|B) = \frac{\text{Prob}(B|A)\text{Prob}(A)}{\text{Prob}(B)}
\]

\[ f_P(\alpha) = f(\alpha | P) = \frac{(1-\alpha)f(\alpha)}{\int_0^\alpha (1-a)f(a)da} \]
Now $I$ stands for "withdraw in Period 1"

$P$ stands for "did not (or could not) withdraw in Period 1"

$\alpha_1$ measures actually withdraw in period 1

2 Technologies (Wallace & others)

liquid: 1 unit yields 1 unit in period 1

OR $R_l > 1$ units in period 2

illiquid: 1 unit yields 0 in period 1

but $R_i$ units $> R_l > 1$ in period 2
Sequential Service: \( z \) "order" in queue

Wallace:

\( c^1(z) : \) withdrawal in period 1 (by impatient and possibly by runners)
Lower case $c'$'s introduced for study of Glass-Steagall bank

$c^1(z)$: period-1 withdrawal

$c^2_I(\alpha_1)$: period-2 withdrawal "from liquid asset" by period-1 withdrawer

$c^2_P(\alpha_1)$: period-2 withdrawal "from liquid asset" by period-1 non-withdrawer

$\gamma$: fraction of $y$ invested in $l$

$C^2_I(\alpha_1) = c^2_I(\alpha_1) + (1 - \gamma)R_iy$.

$C^2_P(\alpha_1) = c^2_P(\alpha_1) + (1 - \gamma)R_iy$.

$C^2_I(\alpha_1) \geq 0$ and $C^2_P(\alpha_1) \geq 0$
\[ \alpha_1 c_f^2(\alpha_1) + (1 - \alpha_1) c_P^2(\alpha_1) = [\gamma y - \int_0^{\alpha_1} c^1(z) \, dz] R_\ell. \quad \text{(RC)} \]

\[ M = \{ \gamma, c^1(z), c_f^2(\alpha_1), c_P^2(\alpha_1) \mid \text{Equation (RC) holds for all } \alpha_1 \}. \]
2 Financial Systems:

- Unrestricted Bank, Unified System
- Restricted Bank, Glass-Steagall Bank, Separated System
(1) In the *separated financial system*, consumers place a fraction \((1 - \gamma)\) of their wealth in technology \(i\), whose return cannot be touched by the bank. In terms of resource constraint (RC), this is equivalent to imposing the additional constraints: \(c^2_P(\alpha_1) \geq 0\) and, more importantly, \(c^2_I(\alpha_1) \geq 0\). Combined with incentive compatibility, these additional constraints give rise to overinvestment in technology \(\ell\) and the possibility of bank runs.
(2) In the *unified financial system*, the bank is able to invest in both technologies. This allows the bank more flexibility in smoothing consumption and preventing runs. For example, when \( \bar{a} \) consumers arrive in period 1 (the worst case scenario), the bank can liquidate all of its technology \( \ell \) holdings, but differentially reward consumers from technology \( i \) in period 2. Consumers who arrive in period 1 might receive less than \( (1 - \gamma)R_iy \) in period 2, while consumers who wait might receive more than \( (1 - \gamma)R_iy \). In terms of resource constraint (RC) this is equivalent to allowing \( c_P^2(\alpha_1) \) or \( c_I^2(\alpha_1) \) to be negative.
Sequential Service Result

\[ c^1(z) = 1 \quad \text{for} \quad z \leq \gamma y. \]
\[ c^1(z) = 0 \quad \text{otherwise} \]
Welfare under Unrestricted Banking

\[ W = \int_{0}^{\gamma y} [\bar{u} + (1 - \alpha)u((1 - \gamma)yR_A + c_P^2(\alpha) - 1) + \alpha u((1 - \gamma)yR_A + c_I^2(\alpha))] f(\alpha) d\alpha \]

\[ + \int_{0}^{\bar{\alpha}} [(1 - \alpha + \gamma y)\bar{u} + (\alpha - \gamma y)\beta \bar{u} + (1 - \alpha)u((1 - \gamma)yR_A + c_P^2(\alpha) - 1) + (\alpha - \gamma y)u((1 - \gamma)yR_A + c_I^2(\alpha) - 1) + \gamma y u((1 - \gamma)yR_A + c_I^2(\alpha)) f(\alpha) d\alpha] \]
Incentive Compatibility (ICC):
$I$ denotes early withdrawal (running), $P$ denotes early non-withdrawal

$$\int_0^{\bar{\alpha}} u(c_P^2(\alpha)) + (1 - \gamma)yR_A - 1) f_P(\alpha) d\alpha$$

$$\geq \int_0^{\gamma y} u(c_I^2(\alpha)) + (1 - \gamma)yR_A) f_P(\alpha) d\alpha$$

$$+ \int_{\gamma y}^{\bar{\alpha}} (\gamma y / \alpha) u(c_I^2(\alpha)) + (1 - \gamma)yR_A)$$

$$+(1 - \gamma y / \alpha) u(c_P^2(\alpha)) + (1 - \gamma)yR_A - 1) f_P(\alpha) d\alpha.$$ \hspace{1cm} (5)

Resource constraint (RC):

$$\alpha_1 c_I^2(\alpha_1) + (1 - \alpha_1) c_P^2(\alpha_1) = (\gamma y - \alpha_1)R_B \quad \text{if} \ \alpha_1 \leq \gamma y$$

$$\gamma yc_I^2(\alpha_1) + (1 - \gamma y)c_P^2(\alpha_1) = 0 \quad \text{if} \ \alpha_1 > \gamma y.$$ \hspace{1cm} (6)
Profit-maximizing perfectly-competitive bank chooses the contract so as to:

\[
\begin{align*}
\max W \\
\text{wrt } \gamma, c_i^2(\alpha_1), c_P^2(\alpha_1) \\
\text{subject to ICC(5) and RC (6).}
\end{align*}
\]
The so-called "optimal contract" for the unified system satisfies $\gamma y < \bar{\alpha}$. The "first" $\gamma y$ impatient consumers to arrive are fully served by the bank in period 1. There is a positive probability that $\alpha > \gamma y$ holds, in which case $(\alpha - \gamma y)$ impatient consumers are rationed. Patient consumers do not withdraw in period 1, and we have full consumption smoothing, i.e.,

$$c^2_i(\alpha_1) = c^2_p(\alpha_1) - 1$$

for all $\alpha_1 \leq \gamma y$.  \hspace{1cm} (8)
Proof:

\[
\left( \frac{\partial W}{\partial \gamma} \right)_{\gamma=\bar{\alpha}/y} = \int_0^{\bar{\alpha}} y(R_B - R_A) u'[c^2_P(\alpha) - 1 + (y - \bar{\alpha})R_A] f(\alpha) d\alpha < 0.
\]

Consumption Smoothing:

\[
c^2_P(\alpha_1) = c^2_l(\alpha_1) + 1 \quad \text{for all } \alpha_1,
\]

So \( C^2_P(\alpha_1) = C^2_l(\alpha_1) + 1 \) \( \text{for all } \alpha_1 \)

\( c^2_l(\alpha_1) \) could be negative, even though \( C^2_P(\alpha_1) \) and \( C^2_l(\alpha_1) \) must be non-negative
Central Result for Unified System:

- Under the optimal contract, there is no run equilibrium.
The Separated System (Restricted or Glass-Steagall Bank) holds only /

\( \gamma y \) deposited in bank

\( (1 - \gamma)y \) deposited in mutual fund with gross return \( R_i \), which is perfectly illiquid

Glass-Steagall Constraint (GSC):

\[ c_i^2(\alpha_1) \geq 0 \quad \text{and} \quad c_p^2(\alpha_1) \geq 0 \quad \text{for all} \quad \alpha_1. \] (10)
Bank’s Problem

\[ \max W \]

wrt \( \gamma, c_i^2(\alpha_1), c_P^2(\alpha_1) \) \hspace{1cm} (11)

subject to RC, ICC, and GSC.

\[ \max W(\text{rest. bank}) \leq \max W(\text{unrest. bank}) \]
Glass-Steagall Bank

\[ c_P^2(1) < c_P^2(\bar{\alpha}) < 1 \]
Hence: always a run equilibrium
2 Systems

Unrestricted:

- More stable: never has panic-based runs
- Can run out of cash in period 1 (not a bad thing)

Restricted

- Less stable: always subject to runs
- Only runs out of cash during panic-based runs
Sunspot-driven runs on the Glass-Steagall Bank

Run equilibrium is not an equilibrium to the pre-deposit game

Hence introduce sunspot-triggered runs, which occur with probability $\pi$

Let $W^*$ be welfare under the so-called optimal contract without a run.

Let $\overline{W}$ be welfare under the best contract that is immune to runs.

Let $\underline{W}$ be welfare during a run under the so-called ”optimal contract”. 
Optimal welfare is achieved by risking run is $\pi$ is small.

Red indicates best $W$ as function of $\pi$. If run risk is less than $\pi^*$, employ so-called ”optimal contract”. Otherwise, choose best contract immune to runs.
Numerical Example: The unified system

\[ y = 10, \ u(c) = 100 \log(c) - 249, \ \bar{u} = 20, \ R_A = 1.1, \ \beta = 0.7, \]

uniform distribution with \( \bar{\alpha} = 0.5 \):

\[ f(\alpha) = \begin{cases} 
2 & \text{for } \alpha \in [0, 0.5] \\
0 & \text{otherwise.} 
\end{cases} \quad (9) \]

- \( R_B = 1.05 \), we have \( \gamma = 0.04544 \), \( \gamma_y = 0.4544 \) and \( W = 0.8942 \)
- \( R_B = 1.08 \), we have \( \gamma = 0.04807 \), \( \gamma_y = 0.4807 \) and \( W = 0.9599 \).
Numerical Example: The separated system (The Glass-Steagall Bank)

- $R_B = 1.08$. $\gamma = 0.09445 > 0.04807$, $W = 0.8688 < 0.9599$
- $\gamma$ in GS is about twice $\gamma$ in unrestricted bank.
- high $\gamma$ is anti-growth
Sunspots and Glass-Steagall Example

- $R_B = 1.08$.
- best $\gamma$ to avoid runs is $\bar{\gamma} = 0.09630$, $\pi^* = .5521\%$
- "Runs" back in the bank runs literature.