Lecture Notes #5
IDEAS

Ideas $\rightarrow$ Non-rivalry $\rightarrow$ IRS

$\rightarrow$ Non-competitive $\rightarrow$ Public Financed $\rightarrow$ Imperfect Competition (including Patents)

Excludability - depends in part on transactions-cost
Romer Model


\[
Y = K^\alpha (ALY)^{1-\alpha} \quad \dot{K} = s_K Y - \mu K \quad \dot{L} = nL \quad LA + LY = L
\]

\[
\dot{A} = \theta A^\phi L^\lambda_A \quad \phi > 0 : \text{standing on shoulders}
\]

\[
\lambda < 1 : \text{stepping on toes}
\]
Steady State

\[ g_y = g_k = g_A \]

\[ g_A = \frac{\dot{A}}{A} = \frac{\theta L_A^\lambda}{A^{1-\phi}} = \text{const} \]

Log-differentiating RHS yields: \( \lambda \frac{\dot{L}_A}{L_A} - (1 - \phi) \frac{\dot{A}}{A} = 0 \)

\( \lambda n - (1 - \phi) g_A = 0 \)

\[ g_A = \frac{\lambda n}{(1-\phi)} \]
Special Case #1

\( \lambda = 1 \) and \( \phi = 0 \)

Productivity of researchers = const \( \theta \)

Hence: \( \dot{A} = \theta L_A \quad g_A = n \)
Special Case #2

\[ \lambda = 1 \text{ and } \phi = 1 \]

\[ \dot{A} = \theta L_A A \quad \frac{\dot{A}}{A} = \theta L_A \]

too rapid? See Shell & Romer
Final Goods Sector

\[ Y = L_Y^{1-\alpha} \int_0^A x_j^\alpha \, dj \quad p_Y = 1 \]

Competitive Profit Maximizing Producers (of \( Y \))

\[ \max_{L_Y, x_j} L_Y^{1-\alpha} \int_0^A x_j^\alpha \, dj - wL_Y - \int_0^A p_j x_j \, dj \]

\( p_j \) is the rental price for capital good \( j \) and \( w \) is the wage rate.

FOC: \( w = \frac{(1-\alpha)Y}{L_Y} \quad p_j = \alpha L_Y^{1-\alpha} x_j^{\alpha-1} \)
Intermediate Goods Sector

Firm $j$ purchases at a fixed cost the patent to produce capital of type $j$ from the new output $Y$ on one-to-one basis

Firm $j$ problem: $\max_{x_j} \pi_j = p_j(x_j)x_j - rx_j$

FOC (by symmetry): $p'(x)x + p(x) - r = 0$

$$p = \frac{1}{1+p'(x)x/p} r \text{ or } p = \frac{r}{\alpha}$$

$\frac{1}{\alpha}$ is the mark-up over MC, $r$
Intermediate Goods Sector

\[ x_j = x \quad \pi_j = \pi \quad \pi = \alpha(1 - \alpha) \frac{Y}{A} \]

Supply of capital = demand for capital:

\[ \int_0^A x_j^\alpha \, dj = K \text{ or } x = \frac{K}{A} \]

Using \( x_j = x \), we have:

\[ Y = AL_Y^{1-\alpha} x^\alpha = AL_Y^{1-\alpha} A^{-\alpha} K^\alpha = K^\alpha (AL_Y^{1-\alpha}) \]
Research Sector

Asset-Market Clearing Equation

Owner of 1 unit of $K$ earns $r$: There is ”no capital gain on $K” because $p_Y = 1$

Capitalist could also buy a patent at price $p_A$ earning profit $\pi/p_A$ and expecting the capital gain $\dot{p}_A/p_A$

Asset markets do not clear unless: $(\pi + \dot{p}_A)/p_A = r$

Quasi-rents plus capital gains per dollar are equalized.

Assuming perfect foresight
In balanced growth: \( p_A = \frac{\pi}{r-n} \)

This is the price of a patent on the balanced growth path.

Labor market: \[ w_Y = \frac{(1-\alpha)Y}{L_Y} \quad w_R = MP_{L_A} \times p_A \]

In equilibrium: \[ w_Y = w_R = w \]

Hence (after calculations) we have:

\[ s_R = \frac{1}{1 + \frac{r-n}{\alpha g_A}} \quad r = \alpha^2 Y / K \quad MP_K = \alpha Y / K \quad r < MP_K \]