Taxes and Transfers Denominated in Money: Static Case


$$\max_{x_h} u_h(x_h)$$

$$s, t, \quad p \cdot x_h = p \cdot \omega_h - p^m \tau_h$$

$$p = (p^1, ..., p^i, ..., p^l) \in R^l_{++}, \quad p^1 = 1$$

$$x_h \in R^l_{++}, \quad \omega_h \in R^l_{++}, \quad \tau_h \in R, \quad p^m \in R_+$$
$n$ individuals:  $h = 1, 2, \ldots n$

a fiscal policy $\tau = (\tau_1, \ldots, \tau_h, \ldots \tau_n) \in \mathbb{R}^n$ is said to be balanced if $\sum_{h=1}^{n} \tau_h = 0$

a fiscal policy $\tau = (\tau_1, \ldots, \tau_h, \ldots \tau_n) \in \mathbb{R}^n$ is said to be bonafide if there is an equilibrium in which $p^m > 0$
Equilibrium:

\[(p, p^m) \in R^l_{++} \times R_+ \text{ in which } \sum_{h=1}^n x_h = \sum_{h=1}^n \omega_h\]

Proper Monetary Equilibrium:

\[(p, p^m) \in R^l_{++} \times R_{++} \text{ in which } \sum_{h=1}^n x_h = \sum_{h=1}^n \omega_h\]
**Proposition:** every bonafide $\tau$ is balanced

Proof:

\[ p \cdot x_h = p \cdot \omega_h - p^m \tau_h \]

Summing over $h$ yields:

\[ p \cdot \sum_{h=1}^{n} x_h = p \cdot \sum_{h=1}^{n} \omega_h - p^m \sum_{h=1}^{n} \tau_h \]

In equilibrium: \[ \sum_{h=1}^{n} x_h = \sum_{h=1}^{n} \omega_h \]

Hence: \[ p^m \sum_{h=1}^{n} \tau_h = 0. \] That is $p^m = 0$ or $\sum_{h=1}^{n} \tau_h = 0$
**Proposition:** for nice utility functions, every balanced fiscal policy is bonafide

Proof:

Tax-adjusted endowment $\tilde{\omega}_h = (\tilde{\omega}_1^h, ..., \tilde{\omega}_l^h)$.

$\tilde{\omega}_i = \omega_i^1 - p^m \tau_h$, $\tilde{\omega}_i = \omega_i^i 
\quad i = 2, ..., l$

$\omega_h = p \cdot \omega_h - p^m \tau_h$

$p \cdot \tilde{\omega}_h = p^1 \omega_1^1 - p^1 p^m \tau_h + p^2 \omega_2^2 + ... + p^l \omega_l^l 
\quad p^1 = 1$

$p \cdot \tilde{\omega}_h = w_h = p \cdot \omega_h - p^m \tau_h$
\[ \tau_h < 0 \Rightarrow \tilde{\omega}_h \in R^l_{++} \]

\[ \tau_h > 0 \Rightarrow \text{for small } p^m, \tilde{\omega}_h \in R^l_{++} \]
Extension to 2 periods

\[ \omega_h = (\omega_h^{1,1}, \ldots, \omega_h^{1,l}, \omega_h^{2,1}, \ldots, \omega_h^{2,l}) \in \mathbb{R}^{2l}_{++}, \quad x_h \in \mathbb{R}^{2l}_{++}, \quad p \in \mathbb{R}^{2l}_{++}, \]

\[ \max_{x_h} u_h(x_h) \]

\[ s, t, \quad p^1 \cdot x_h^1 + p^2 \cdot x_h^2 + p^{m,1} x_h^{m,1} + p^{m,2} x_h^{m,2} = p^1 \cdot \omega_h^1 + p^2 \cdot \omega_h^2 \]

\[ -p^{m,1} \tau_h^1 - p^{m,2} \tau_h^2 \]

\[ x_h^{m,1} + x_h^{m,2} = 0 \quad \text{so} \quad x_h^{m,1} = -x_h^{m,2} \]
Present Price of Money is Constant

\[ p^{m,1} = p^{m,2} = p^m \in R_+ \]

Proof:

We have \( p \cdot x_h + (p^{m,1} - p^{m,2})x^{m,1}_h = p \cdot \omega_h - p^{m,1}\tau^1_h - p^{m,2}\tau^2_h \)

If \( p^{m,1} > p^{m,2} \) then consumers will choose \( x^{m,1}_h \) very negative to get unbounded utility. Sell High, Buy Low.

If \( p^{m,1} < p^{m,2} \) then consumers will choose \( x^{m,1}_h \) to be very large.

Buy Low, Sell High!

\[ x^{m,t}_h > 0 \quad h \text{ lends at } t. \quad x^{m,t+1}_h < 0 \quad h \text{ borrows at } t. \]
Ricardo and budget constraint

\[ p \cdot x_h = p \cdot \omega_h - p^m \tau^1_h - p^m \tau^2_h = p \cdot \omega_h - p^m \mu_h \]

\[ \mu_h = (\tau^1_h + \tau^2_h) \]

Timing of taxes foes not matter, only their present value
International finance

2 consumers, $R$ for red, and $B$ for blue. Static $I = 1$

\[ px_h = p \omega_h - p^R \tau^R_h - p^B \tau^B_h \]

\[ p \sum x_h = p \sum \omega_h - p^R \sum \tau^R_h - p^B \sum \tau^B_h \]

\[ p^R \sum \tau^R_h + p^B \sum \tau^B_h = 0 \]

Bonafide if $\sum \tau^R_h = \sum \tau^B_h = 0$

Bonafide if $p^R \sum \tau^R_h + p^B \sum \tau^B_h = 0$

Assume $\sum \tau^R_h \neq 0$, then for $p^R$ and $p^B$ to be positive

\[ \text{sign}(\sum \tau^R_h) \neq \text{sign}(\sum \tau^B_h) \]
Exchange rate determined by

\((p^R / p^B) = -\left( \sum \tau^B_h / \sum \tau^R_h \right)\)

Units. \( R = $, B = €, x = \text{chocolate} \)

\( p^R = \text{chocolates/$} \quad p^B = \text{chocolates/€} \)

\( \tau^R_h = $ \quad p^B = € \)

\( \frac{\text{chocolates/$}}{\text{chocolates/€}} = \frac{€}{$} \)

\( \frac{€}{$} = \frac{€}{$} \)
Overlapping Generations

Example: 1 person per generation 2 periods 1 commodity per period

\[ \omega_0 = \omega^1_h = 1 \quad x_0 = x^1_0 \]

\[ \omega_t = (\omega^t_t, \omega^{t+1}_t) = (1, 1) \quad x_t = (x^t_t, x^{t+1}_t) \quad \text{for } t = 1, 2, ... \]

\[ p = p^1, p^2, ..., p^t, p^{t+1}, ..., \quad p^{m,t} = p^m \geq 0 \]

\[ px_h = p \omega_h - p^R \tau^R_h - p^B \tau^B_h \]

\[ p \sum x_h = p \sum \omega_h - p^R \sum \tau^R_h - p^B \sum \tau^B_h \]

\[ p^R \sum \tau^R_h + p^B \sum \tau^B_h = 0 \]

Bonafide if \( \sum \tau^R_h = \sum \tau^B_h = 0 \)
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\[ u_0(x_0) = u_0(x_0^1) = x_0^1 \quad u_t(x_t) = u_t(x_t^t, x_t^{t+1}) = x_t^t + x_t^{t+1} \]

Non-monetary CE is autarky

\[ x_t = \omega_t \quad t = 0, 1, \ldots \quad \text{with } p^t = p^1 = 1 \quad t = 1, 2, \ldots \]

Interest rate = 0 \quad CE is not PO. Over-saving!
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Chain Letter ! Ponzi Game

Infinite Horizon allows for CE which are not PO,
even though they are WPO and SRPO. Hilbert’s Hotel

Money and OG

Monetary reform!

\((\tau_t + \tau_{t+1})\) can make everyone better off than in non-monetary CE