Lecture Notes #9

"Lump-Sum Taxes and Transfers: Public Debt in the Overlapping-Generations Model" (with Yves Balasko) in Essays in Honor of Kenneth J. Arrow, Vol. II: Equilibrium Analysis (W. Heller, R. Starr, and D. Starrett, eds.), New York: Cambridge University Press, 1986, Chapter 5, 121-153.

Bonafidelity versus Balancedness

As you have seen, a key is the consumption set. For example,

if each x_h is in the positive orthant then taxes might be

too large for some h so that there is no "equilibrium"

in which h's income is positive. Then Mr h can not "survive"

In finite economies, if utility functions are nice:

Bonafide if and only if Balanced

If utility functions are not nice but the economy is

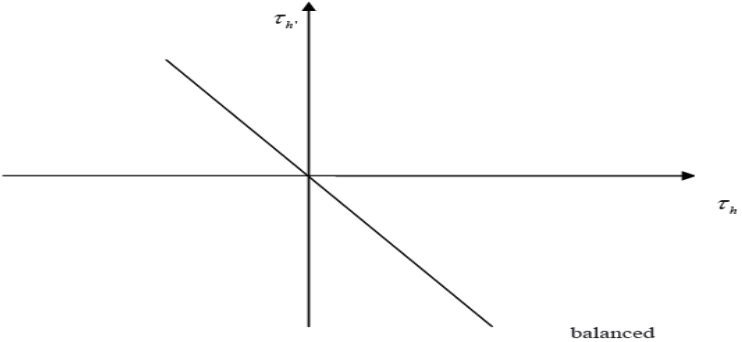
"irreducible" (McKenzie) or "productively related "(Arrow &Hahn)

then bonafide $\Leftrightarrow$ balanced

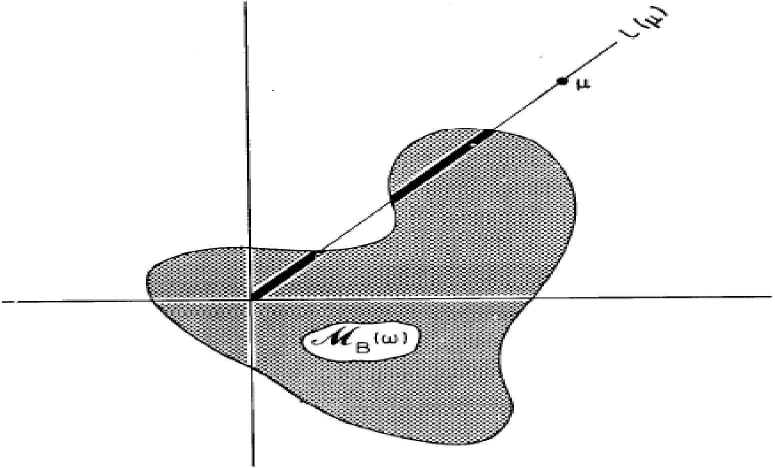
Finite Economy

$\tau = (\tau_{1,\ldots}\tau_{h,\ldots,}\tau_n) \in R^n$ is normalized bonafide if there is

an equilibrium with $p^m = 1$



$n > 2$



OG T=∞

Ricardo: "Public Debt Must be retired"

Samuelson: Counterexample in OG. Carries over to general

open horizon, T=∞

Definition: $\tau = (\tau_{1,\dots}\tau_{t,\dots}) \in R^{\infty}$ is strongly balanced if there is

$t^{'} \in \{0, 1, \dots\}$ with the property that $\sum_{s=0}^{t} \tau_s = 0$ for $t \geq t'$

Proposition: If τ is strongly balanced, then τ is bonafide

Remark: If τ is strongly balanced, then $\tau_t = 0$ for $t > t'$

"Conjecture" If $\tau \in R^{\infty}$ is asymptotically balanced, then τ is bonafide

asymptotically balanced $\Rightarrow \lim_{t \to \infty} \sum_{s=0}^{t} \tau_s = 0$

Counterexample to "Conjecture"

$\tau = (\tau_{0,\ldots}\tau_{t,\ldots}) = -(\mu_{0,\ldots}\mu_{t,\ldots})$ where $\tau_0 = \tau_0^1$, $\tau_t = \tau_t^t + \tau_t^{t+1}$ for $t = 1, 2,$

$\mu_0 = 1/(1-\delta)$, $\mu_t = -\delta^{t-1}$ for $t = 1, 2, \ldots 0 < \delta < 1$

$$\lim_{t\to\infty} \sum_{s=0}^{t} \mu_s = 1/(1-\delta) - 1/(1-\delta) = 0$$

$\sum_{s=0}^{t} \mu_s > 0$, for $t = 1, 2, ..$

$\omega_t = (\omega_t^t, \omega_t^{t+1}) = (a, b) \in R_{++}^2$ for $t = 1, 2, ..$

$$w_t = p^m \mu_t + p^t \omega_t^t + p^{t+1} \omega_t^{t+1} = -p^m \delta^{t-1} + p^t a + p^{t+1} b$$

$$\widetilde{\omega}_t = (\widetilde{\omega}_t^t, \widetilde{\omega}_t^{t+1}) = (\omega_t^t - p^m \delta^{t-1}/p^t, \omega_t^{t+1}) = (a - p^m \delta^{t-1}/p^t, b)$$

$$x_t^t = a - \frac{p^m}{p^t}\left[\frac{1}{1-\delta} - \sum_{s=0}^{t-2}\delta^s\right]$$

$$\sum_{s=0}^{t}\mu_s = \frac{1}{1-\delta} - \sum_{s=0}^{t-1}\delta^s = \left[\frac{1}{1-\delta} - \sum_{s=0}^{t-2}\delta^s\right] - \delta^{t-1} > 0$$

$$\Rightarrow \frac{1}{1-\delta} - \sum_{s=0}^{t-2}\delta^s > \delta^{t-1}$$

$$\Rightarrow \omega_t^t > \widetilde{\omega}_t^t > x_t^t$$

Assume $\sup_{(x_t^t, x_t^{t+1}) \in \pi} \left(\frac{\partial u_t / \partial x_t^{t+1}}{\partial u_t / \partial x_t^t}\right) < \delta - \epsilon, \epsilon > 0$

$$\Rightarrow 0 < \left(p^{t+1}/p^t\right) < \delta - \epsilon$$

$$\Rightarrow \widetilde{\omega}_t^t < a - p^m \delta^{t-1}/(\delta - \epsilon)^{t-1}$$

For t large, $\widetilde{\omega}_t^t < 0$ which contradicts $x_t^t > 0$