Bonafidelity versus Balancedness

As you have seen, a key is the consumption set. For example, if each $x_h$ is in the positive orthant then taxes might be too large for some $h$ so that there is no "equilibrium" in which $h$’s income is positive. Then Mr $h$ cannot "survive"
In finite economies, if utility functions are nice:

Bonafide if and only if Balanced

If utility functions are not nice but the economy is

"irreducible" (McKenzie) or "productively related" (Arrow & Hahn)

then bonafide $\iff$ balanced
Finite Economy

\[ \tau = (\tau_1, \ldots, \tau_h, \ldots, \tau_n) \in R^n \] is normalized bonafide if there is an equilibrium with \( p^m = 1 \).
\[ n > 2 \]
Ricardo: ”Public Debt Must be retired”

Samuelson: Counterexample in OG. Carries over to general open horizon, \( T = \infty \)

Definition: \( \tau = (\tau_1, \ldots, \tau_t, \ldots) \in R^\infty \) is strongly balanced if there is

\[ t' \in \{0, 1, \ldots\} \] with the property that \( \sum_{s=0}^{t} \tau_s = 0 \) for \( t \geq t' \)
Proposition: If \( \tau \) is strongly balanced, then \( \tau \) is bonafide

Remark: If \( \tau \) is strongly balanced, then \( \tau_t = 0 \) for \( t > t' \)

"Conjecture" If \( \tau \in R^\infty \) is asymptotically balanced, then \( \tau \) is bonafide

asymptotically balanced \( \Rightarrow \lim_{t \to \infty} \sum_{s=0}^{t} \tau_s = 0 \)
Counterexample to "Conjecture"

\[ \tau = (\tau_0, \ldots, \tau_t, \ldots) = -(\mu_0, \ldots, \mu_t, \ldots) \] where \( \tau_0 = \tau_0^1, \tau_t = \tau_t^t + \tau_t^{t+1} \) for \( t = 1, 2, \ldots \)

\[ \mu_0 = \frac{1}{1 - \delta}, \mu_t = -\delta^{t-1} \] for \( t = 1, 2, \ldots \quad 0 < \delta < 1 \)

\[ \lim_{t \to \infty} \sum_{s=0}^{t} \mu_s = \frac{1}{1 - \delta} - \frac{1}{1 - \delta} = 0 \]

\[ \sum_{s=0}^{t} \mu_s > 0, \text{ for } t = 1, 2, \ldots \]

\[ \omega_t = (\omega_t^t, \omega_t^{t+1}) = (a, b) \in \mathbb{R}^2_{++} \] for \( t = 1, 2, \ldots \)

\[ w_t = p^m \mu_t + p^t \omega_t^t + p^{t+1} \omega_t^{t+1} = -p^m \delta^{t-1} + p^t a + p^{t+1} b \]

\[ \tilde{\omega}_t = (\tilde{\omega}_t^t, \tilde{\omega}_t^{t+1}) = (\omega_t^t - p^m \delta^{t-1} / p^t, \omega_t^{t+1}) = (a - p^m \delta^{t-1} / p^t, b) \]
\( x_t^t = a - \frac{p^m}{p_t} \left[ \frac{1}{1-\delta} - \sum_{s=0}^{t-2} \delta^s \right] \)

\[
\sum_{s=0}^{t} \mu_s = \frac{1}{1-\delta} - \sum_{s=0}^{t-1} \delta^s = \left[ \frac{1}{1-\delta} - \sum_{s=0}^{t-2} \delta^s \right] - \delta^{t-1} > 0
\]

\[
\Rightarrow \frac{1}{1-\delta} - \sum_{s=0}^{t-2} \delta^s > \delta^{t-1}
\]

\[
\Rightarrow \omega_t^t > \tilde{\omega}_t^t > x_t^t
\]

Assume \( \sup_{(x_t^t, x_t^{t+1}) \in \pi} \left( \frac{\partial u_t}{\partial x_t^{t+1}} / \frac{\partial u_t}{\partial x_t^t} \right) < \delta - \epsilon, \epsilon > 0 \)

\[
\Rightarrow 0 < \left( \frac{p^{t+1}}{p^t} \right) < \delta - \epsilon
\]

\[
\Rightarrow \tilde{\omega}_t^t < a - p^m \delta^{t-1} / (\delta - \epsilon)^{t-1}
\]

For \( t \) large, \( \tilde{\omega}_t^t < 0 \) which contradicts \( x_t^t > 0 \)