Political Economy of Price-Level Volatility: Incomplete Money-Tax Instruments & Incomplete Market-Participation

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Adam Smith (1776): A prince who should enact that a certain proportion of his taxes should be paid in a paper money of a certain kind might thereby give a certain value to this paper money.
Taxes Denominated in Money

- 3 consumers (or 3 types): \( h = 1, 2, 3 \).

- Lump-Sum dollar taxation: \( \tau = (\tau_1, \tau_2, \tau_3) \in \mathbb{R}^3 \) with \( \tau \) balanced so that \( \tau_1 + \tau_2 + \tau_3 = 0 \). Hence, \( \tau = (\tau_1, \tau_2, -(\tau_1 + \tau_2)) \) is a 2-dimensional policy tool.

- Dollar tax based on income: \( \tau = (\tau_1, \tau_2, \tau_3) \in \mathbb{R}^3 \) but is restricted by

\[
\tau_h = \theta \omega_h - \theta \bar{\omega},
\]

where \( \theta \geq 0 \) is the tax rate, \( \omega_h \) is Mr. \( h \)'s ex-ante endowment of chocolate, and \( \bar{\omega} = (\omega_1 + \omega_2 + \omega_3) / 3 \) is the mean endowment. Hence \( \theta \) is a 1-dimensional policy tool. Notice that \( \tau_1 + \tau_2 + \tau_3 = 0 \).

- Bhattacharya, Guzman, & Shell (1998).
  - Fully anticipated exogenous lump-sum \( \tau \)-taxation denominated in dollars.
Balanced and Bonafide Taxes

- \( x_h = \omega_h - P^m \tau_h \).

- Summing over \( h \),
  \[ \sum_h x_h = \sum_h \omega_h - P^m \sum_h \tau_h. \]

- Hence, Ricardo (in finite economy):
  - Either \( \tau \) is balanced or \( P^m = 0 \) or both
  - Balasko-Shell bonafidelity
Endogenous Money Taxes

- Fully Anticipated, Endogenous Money Taxation
  - $\tau$-Planner
  - $\theta$-Planner
  - $\theta$-Voting
  - Compare with costly commodity taxation.

- Goal
  - Analyze effects of Fully Anticipated Volatility on Equilibrium Taxes and Allocations
  - Real taxation is relatively inefficient in good times, but it does not create excess volatility.
  - Money taxation is relatively efficient, but does permit excess volatility.
• Hints that Price-Level Volatility is bad:
  • Friedman
  • Shiller
  • Cass-Shell Immunity Theorem
• When Volatility is good:
  • Prescott-Townsend
  • Shell-Wright
• Winners & Losers from volatility:
  • Focus of this talk
Time Line

- Expectations formed
- Taxes chosen
- Securities traded
- Taxes collected & securities paid
- Consumption
• 2 states: \( s = \alpha, \beta \).
  • \( \pi(\alpha) + \pi(\beta) = 1 \).

• \( s \) can be interpreted as:
  • intrinsic uncertainty (random endowments, or monetary trembles), or
  • extrinsic uncertainty (“sunspots”).

• Restriction #1: Incomplete Instruments:
  • \( \tau_h(\alpha) = \tau_h(\beta) = \tau_h \).

• Restriction #2: Incomplete Participation:
  • Mr. 1 & Mr. 2 can trade on the securities market and the spot market. Their participation is unrestricted.
  • Mr. 3 can only trade on the spot market. He cannot trade on the securities market. His participation is restricted.
• There are two problems of multiplicity/indeterminacy.
  1. Given the real (commodity) values of taxes and transfers, there may be multiple equilibria in the spot market.
  2. There may be indeterminacy of the price level.

• We want to focus on the effect of the second (price-level volatility) on the choice of taxes and transfers.

• Thus, we choose to work with log-linear preferences, precluding the first multiplicity.
Specifications

• \( X_h = \mathbb{R}^2_{++} \).

• \( \pi(\alpha) = \pi(\beta) = \frac{1}{2} \).

• \( V_h(x_h(\alpha), x_h(\beta)) = \frac{1}{2} \log(x_h(\alpha)) + \frac{1}{2} \log(x_h(\beta)), \ h = 1, 2, 3. \)

• \( \omega_h(\alpha) = \omega_h(\beta) = \omega_h. \)

• Ex-ante chocolate is the numeraire.

• \( p(s) \) is the price of chocolate in state \( s = \alpha, \beta. \)

• \( p^m(s) \) is the price of money in state \( s = \alpha, \beta. \)

• \( P^m(s) = p^m(s) / p(s) \) is the chocolate price of money (the inverse of the price level) in state \( s. \)

• \( \tilde{\omega}_h(s) = \omega_h - P^m(s) \tau_h \) is the tax adjusted endowment in state \( s. \)
Specifications (Continued)

- Base case: $P^m(\alpha) = P^m(\beta) = P^m = 10$.
- Mean-Preserving Spreads:
  - $P^m(\alpha) = P^m - \sigma = 10 + \sigma$.
  - $P^m(\beta) = P^m + \sigma = 10 + \sigma$

where the standard deviation $\sigma$ is non-negative.
Consumer Problems

- Restricted consumer 3 chooses \( x_3(s) \in \mathbb{R}_{++} \) to maximize

\[
\log(x_3(s))
\]

subject to

\[
p(s)x_3(s) = p(s) \omega_3 - p^m(s) \tau_3, \quad s = \alpha, \beta.
\]

- The budget constraints for Mr. 3 can be written as

\[
x_3(s) = \tilde{\omega}_3(s), \quad s = \alpha, \beta.
\]

- Mr. 3 consumes his tax-adjusted endowment. (Formally, there are two Mr. 3’s: Mr. 3\( \alpha \) and Mr. 3\( \beta \).)
• Unrestricted consumer $h = 1, 2$ chooses
$(x_h(\alpha), x_h(\beta)) \in \mathbb{R}^2_{++}$ to

$$\text{maximize} \quad \frac{1}{2} \log(x_h(\alpha)) + \frac{1}{2} \log(x_h(\beta))$$

subject to

$$p(\alpha)x_h(\alpha) + p(\beta)x_h(\beta) = (p(\alpha) + p(\beta))\omega_h - (p^m(\alpha) + p^m(\beta))\tau_h.$$ 

so that

$$p(\alpha)x_h(\alpha) + p(\beta)x_h(\beta) = p(\alpha)\tilde{\omega}_h(\alpha) + p(\beta)\tilde{\omega}_h(\beta).$$

• Connection between extrinsic uncertainty & intrinsic uncertainty
  • Edgeworth box is now a proper rectangle, not a square.
Competitive Equilibrium

- A **competitive equilibrium** is a price vector 
  \( (p(\alpha), p(\beta), p^m(\alpha), p^m(\beta)) \) with \( p(s) > 0 \) and \( p^m(s) \geq 0 \) for \( s = \alpha, \beta \) with demand equal supply,

  \[
  \sum_{h=1,2,3} x_h(s) = \sum_{h=1,2,3} \omega_h, \text{ for } s = \alpha, \beta.
  \]

- Summing over the unrestricted consumers’ budget constraints yields

  \[
  p(\alpha) \sum_{h=1,2} x_h(\alpha) + p(\beta) \sum_{h=1,2} x_h(\beta) = p(\alpha) \sum_{h=1,2} \tilde{\omega}_h(\alpha) + p(\beta) \sum_{h=1,2} \tilde{\omega}_h(\beta).
  \]
The equilibrium behavior of the unrestricted consumers can be described in terms of their state-specific tax-adjusted endowments in the tax-adjusted Edgeworth box of dimensions

$$\sum_{h=1,2} \tilde{\omega}_h(\alpha) \times \sum_{h=1,2} \tilde{\omega}_h(\beta).$$

$$[p^m(\alpha) + p^m(\beta)] \sum_{h=1,2,3} \tau_h = 0.$$

Set

$$P^m(\alpha) = 10 - \sigma \text{ and } P^m(\beta) = 10 + \sigma.$$

The planner is Benthamite, i.e. he chooses $\tau$ or $\theta$ to maximize

$$W = V_1 + V_2 + V_3.$$
The $\tau$-Planner

- Given $\tau_3$, $P^m(\alpha)$, and $P^m(\beta)$, Mr. 3 consumes his tax-adjusted endowments $\bar{\omega}_3(\alpha)$ and $\bar{\omega}_3(\beta)$. 

- Mr. 1 & Mr. 2 trade in the tax-adjusted Edgeworth box. The optimal strategy for the $\tau$-planner is to equalize their marginal rates of substitution, i.e. to set $V_1 = V_2$ (for identical preferences).
  - The $\tau$-planner’s problem is in this case seemingly “1-dimensional”: choosing the lump-sum tax $\tau_3$ for the restricted consumer.
  - In general, the $\theta$-planner is less powerful than the $\tau$-planner: $\max W^{\theta} \leq \max W^{\tau}$. In our examples, the $\tau$-planner has 2 tools while the $\theta$-planner has only one tool. With more heterogeneity, the power of the $\tau$-planner relative to that of the $\theta$-planner is increased.

- In computing the taxes chosen by social planner: fix price level expectations and normalize $p(\alpha) = 1$. Allow $p(\beta)$ to adjust for markets to clear according to demand given the chosen tax.
Propositions and Illustrative Calculations

• Proposition 1: $V_1 = V_2$ under $\tau$-planning.
• Proposition 2: $\max_\tau W \geq \max_\theta W$.
• Proposition 3: When $\sigma = 0$, the $\tau$-planner levels, i.e. he gives everyone the same allocation. The $\theta$-planner also levels when volatility is zero.
• Social welfare function is not necessarily concave in prices, so further theorems even in this simple world do not seem easy to come by.
Some Computations

- Mr. 1 & Mr. 2 are unrestricted.
- Mr. 3 is restricted.
- $P^m(\alpha) = 10 - \sigma$.
- $P^m(\beta) = 10 + \sigma$.
- $\pi(\alpha) = \pi(\beta) = 0.5$.
- $\tau$-Planner in blue.
- $\theta$-Planner in red.
Sample Calculation 1

- Mr. 1 is rich, \( \omega_1 = 80 \).
- Mr. 2 is middle-class, \( \omega_2 = 60 \), and holds the median = the mean endowment.
- Mr. 3 is poor, \( \omega_3 = 40 \), and also disadvantaged in trading.
- \( \bar{\omega} = \frac{80 + 60 + 40}{3} = 60 = \omega_2 = \omega_{med} \).
- When \( \sigma = 0 \), under \( \tau \)-taxation \( x_h(\alpha) = x_h(\beta) = \bar{\omega} \). Same for voting.
Vary $\text{std}(P_m)$ with mean = 10
with $\omega_1=80$ $\omega_2=60$ $\omega_3=40$; $\pi(\alpha) = 0.5$; $\lambda_1=\lambda_2=\lambda_3$.
Vary $\text{std}(P_m)$ with mean = 10
with $\omega_1=80$ $\omega_2=60$ $\omega_3=40$; $\pi(\alpha) = 0.5$; $\lambda_1=\lambda_2=\lambda_3$
Sample Calculation 2

- Mr. 1 is rich, $\omega_1 = 80$.
- Mr. 2 and Mr. 3 have the same endowment, $\omega_2 = \omega_3 = 40$, but Mr. 3 is disadvantaged in trading.
- $\bar{\omega} = 160/3, \omega_{med} = 40$.
- When $\sigma = 0$, under $u_1 = u_2 = u_3$ under $\tau$-taxation.
- Mr. 3 is very “expensive” for the $\tau$-planner: For $\sigma > 0$, $u_2 > u_3$. Furthermore, $u_3$ blue is below $u_3$ red. The planner does not give Mr. 3 more because of his handicap. He gives him less, especially when he can target Mr. 3 by name.
Vary $\text{std}(P_m)$ with mean = 10
with $\omega_1=80 \omega_2=40 \omega_3=40; \pi(\alpha) = 0.5; \lambda_1=\lambda_2=\lambda_3$
Vary std($P^m$) with mean $= 10$ with $\omega_1 = 80$, $\omega_2 = 40$, $\omega_3 = 40$; $\pi(\alpha) = 0.5; \lambda_1 = \lambda_2 = \lambda_3$.
Polar cases: \(\tau\)-Planner

- If everyone is unrestricted then the equilibrium allocations are state symmetric (Cass-Shell Immunity Theorem).
- If everyone is restricted, then only when price expectations are state symmetric, i.e. \(P^m(\alpha) = P^m(\beta)\), are equilibrium allocations state symmetric.
- This is different from exogenous taxes where allocations are necessarily state symmetric.
- The intuition for this is that the planner has incomplete instruments and tries to compensate for potentially differing real taxes and transfers in the two states, but is unable to do so.
Majority voting in the fully restricted economy

- Assume $\omega_1 > \bar{\omega} > \omega_2 > \omega_3$. Mean wealth > median wealth.
- $P^m(\alpha) > P^m(\beta) > 0$.
- Preferred money tax rates $\theta$:

\[
\theta_2^* = \arg \max_{\theta \in \left[0, \frac{1}{P^m(\alpha)}\right]} \left\{ \pi(\alpha) u_2 \left[ \omega_2 - \theta P^m(\alpha) (\omega_2 - \bar{\omega}) \right] + \pi(\beta) u_2 \left[ \omega_2 - \theta P^m(\beta) (\omega_2 - \bar{\omega}) \right] \right\} = \frac{1}{P^m(\alpha)}
\]

\[
\theta_3^* = \arg \max_{\theta \in \left[0, \frac{1}{P^m(\alpha)}\right]} \left\{ \pi(\alpha) u_3 \left[ \omega_3 - \theta P^m(\alpha) (\omega_3 - \bar{\omega}) \right] + \pi(\beta) u_3 \left[ \omega_3 - \theta P^m(\beta) (\omega_3 - \bar{\omega}) \right] \right\}
\]
Voting in the fully restricted economy

- The marginal expected utility of the tax rate is positive for Mr. 2 and Mr. 3:

\[(\bar{\omega} - \omega_h) \sum_{s=\alpha}^{\beta} P^m(s) \pi(s) u'_h \left[ \omega_h - \theta P^m(s) (\omega_h - \tilde{\omega}) \right] > 0,\]

- Equilibrium indirect utility is:

\[\pi(\alpha) u_h(\bar{\omega}) + \pi(\beta) u_h \left[ \omega_h - \frac{P^m(\beta)}{P^m(\alpha)} (\omega_h - \bar{\omega}) \right].\]

- Generically, \(\tilde{\omega} \neq \left[ \omega_h - \frac{P^m(\beta)}{P^m(\alpha)} (\omega_h - \bar{\omega}) \right].\)

- Sunspots matter. The SSE allocation is not a mere randomization over CE except in trivial cases. Contrast this with Cass and Shell (1983).
Voting in the fully restricted economy with real taxation

• Now suppose that taxes are denominated in commodities so that: \( \tilde{\omega}_h = \omega_h - \gamma (\omega_h - \bar{\omega}) \), where \( \gamma \) is the uniform, real, "before-demo-grant" tax rate.

\[
\begin{align*}
\gamma_2^* &= \arg \max_{\gamma \in [0,1]} \{ \pi(\alpha) u_2 [\omega_2 - \gamma (\omega_2 - \bar{\omega})] + \pi(\beta) u_2 [\omega_2 - \gamma (\omega_2 - \bar{\omega})] \} \\
&= \arg \max_{\gamma \in [0,1]} u_2 [\omega_2 - \gamma (\omega_2 - \bar{\omega})] = 1; \\
\gamma_3^* &= \arg \max_{\gamma \in [0,1]} \{ \pi(\alpha) u_3 [\omega_3 - \gamma (\omega_3 - \bar{\omega})] + \pi(\beta) u_3 [\omega_3 - \gamma (\omega_3 - \bar{\omega})] \} \\
&= \arg \max_{\gamma \in [0,1]} u_3 [\omega_3 - \gamma (\omega_3 - \bar{\omega})] = 1;
\end{align*}
\]

• Leveled with no volatility: \((x_h(\alpha), x_h(\beta)) = (\bar{\omega}, \bar{\omega})\).

• No price-volatility.
Voting in the fully restricted economy with costly commodity taxation at the rate $\gamma$

- **Iceberg Cost** (melting of chocolate, spoilage of apples, costly indexed security) of commodity taxation at rate $\delta \in [0, 1)$.
- Tax at rate $\gamma \in [0, 1]$ on real wealth $\omega_h$ with balanced-budget level real lump-sum demo-grant transfers $(1 - \delta)\gamma\bar{\omega}$.
- Tax-adjusted endowments are then:
  \[
  \tilde{\omega}_h = \omega_h - \gamma \omega_h + (1 - \delta) \gamma \bar{\omega} = \omega_h - \gamma \left[ \omega_h - (1 - \delta) \bar{\omega} \right].
  \]
- Median voter will choose full-redistribution through commodity-taxation if spoilage cost $\delta$ is small enough. No volatility.
• Comparing $\theta$-money taxation $\tau$ vs. $\gamma$-commodity taxation:
  • If $\delta > \min \left( \frac{\bar{\omega} - \omega_h}{\omega}, \bar{\delta} \right)$, money taxation is preferred., by Mr. h.
  • If $\delta < \min \left( \frac{\bar{\omega} - \omega_h}{\omega}, \bar{\delta} \right)$, commodity taxation is preferred., by Mr. h.

• The critical spoilage rate $\bar{\delta}$ is increasing in $\left( \frac{P^m(\alpha)}{P^m(\beta)} \right)$. A low inflation rate is favorable for the adoption of money-taxation.

• The critical spoilage rate $\bar{\delta}$ is decreasing in $\pi(\alpha)$. A lower probability of inflation favorable for the adoption of money-taxation.