1. Overlapping Generations

2-period lives.

1 commodity per period, \( \ell = 1 \).

Stationary endowments:

\[
\omega_t^0 = B > 0 \text{ for } t = 0
\]
\[
(\omega_t^t, \omega_t^{t+1}) = (A, B) > 0 \text{ for } t = 1, 2, ...
\]

Stationary preferences:

\[
u_0(x_0^1) = D \log x_0^1 \text{ for } t = 0
\]
\[
u_t(x_t^t, x_{t+1}^{t+1}) = C \log x_t^t + D \log x_{t+1}^{t+1} \text{ for } t = 1, 2, ...
\]

Passive fiscal policy:

\[m_0^1 = 2 \quad m_t^s = 0 \text{ otherwise}
\]

Goods price of money is \( p^m \geq 0 \).

Precisely plot (use graph paper if necessary) the offer curve in excess demand space \((x_t^t - \omega_t^t, x_{t+1}^{t+1} - \omega_{t+1}^{t+1})\) for Mr. \( t \geq 1 \). Plot the reflected offer curve, and analyze the global dynamics for each of the following cases:
(a) $A = 1, B = 2, C = 2, D = 1$
(b) $A = 2, B = 2, C = 1, D = 4$
(c) $A = 8, B = 5, C = 4, D = 6$
(d) $A = 2, B = 8, C = 1, D = 4$

Is there a pattern?

Derive the conditions on the MRS for a "Samuelson" versus a "Classical" economy and relate them to the above?

Let $m_0^1 = -1$ (negative money). Redo all the exercises above. Is there a pattern? What happens to the Samuelson economy when going from positive money to negative money? The classical economy? [Hint: Be sure to plot the FULL reflected offer curve.]

2. Overlapping Generations

Pure exchange, 2-period lives, one consumer per generation.

$$u_0 (x_0^1) = x_0^1$$
$$\omega_0^1 = 1, \quad for \ t = 0,$$

$$u_t (x_t^t, x_{t+1}^t) = x_t^t + x_{t+1}^t$$
$$(\omega_t^t, \omega_{t+1}^t) = (1, 1), \quad for \ t = 1, 2, \ldots$$

Money transfers:

$$m_0^1 = 2, \quad m_1^1 = -1,$$
$$m_1^2 = 1, \quad m_t^t = 0 \ otherwise.$$
(a) What is the non-monetary equilibrium allocation? What are the prices? What are the interest rates?

(b) Derive the reflected offer curve for consumer \( t = 1, 2, \ldots \).

(c) Derive the set of equilibrium money prices.

(d) Draw the phase diagram and show the full evolution of this economy (depending on the price of money).

(e) What is the Pareto optimal allocation associated with the above (money) tax-transfer policy?

(f) Find an alternative tax-transfer policy and associated allocation which is not Pareto optimal but in which everyone is strictly better off than they would be in autarky.

(g) Find an alternative tax-transfer policy and associated allocation which is Pareto optimal and in which everyone is strictly better off than they would be in the non-monetary equilibrium.