Economics 614: Macroeconomics II
Spring 2006
Cornell University
Problem Set #10
Due April 17 (Monday)

1 OG model with production

Each person lives for two periods. People work in the first period, when they are young, retire in the second period, when they are old, and then die off. People consume in both periods of life, and they pay for consumption in the second period by saving in the first period.

We shall refer to the cohort that is born at time t as generation t. Memebers of this generation are young in period t and old in period t+1. Therefore, during period t, the young of generation t overlap with the old of generation t-1. The population growth rate is n and there is no capital depreciation. Each person is endowed with 1 unit of labor and he does not derive utility from leisure.

Preference

$$U_t = \frac{c_{y,t}^{1-\theta} - 1}{1-\theta} + \beta \frac{c_{o,t}^{1-\theta} - 1}{1-\theta},$$

where $\theta > 0$ and $0 < \beta < 1$. Note when $\theta \to 1$, then $\frac{c^{1-\theta}-1}{1-\theta} \to \log c$.

Technology

$$Y_t = AK_t^{\alpha} L_t^{1-\alpha},$$

where $A > 0$ and $0 < \alpha < 1$.

(a) Consider an individual born at time t. Let s_t be the amount he saved in period t. From his life time utility maximization problem, get s_t as a function of wage income w_t and the interest rate r_{t+1}.

What is the range of $\frac{\partial s_t}{\partial w_t}$? How does the sign of $\frac{\partial s_t}{\partial r_{t+1}}$ depend on θ?

(b) Let $k_t = K_t/L_t$. Get the difference equation of k in the competitive equilibrium.

(c) Suppose $\theta = 1$. Then $U_t = \log c_{y,t} + \beta \log c_{o,t}$. Compute the steady state capital labor ratio k^* and analyze the full dynamics.

2 Capital Gains in One-sector model with government debt

Normalize the price of consumption good as 1. Let p and p_B denote the price of capital and bonds, respectively. B is the nominal stock of non-interest bearing government debt. Let n be the population growth rate. No capital depreciation. Let s be a constant saving rate.

$$\begin{aligned}
Y &= F(K, L), \text{ Constant return to scales} \\
C &= (1-s)\left[\max(1,p)\,Y + \dot{p}K + \dot{p}_B B + p_B \dot{B}\right], \\
\dot{K} &= Y - C, \\
\frac{\dot{B}}{B} &= \theta,
\end{aligned}$$

where the parameter $\theta < n$.

(a) Define $k = \frac{K}{L}$ and $b = \frac{p_B B}{L}$. Derive the law of motion $\dot{k}$ and $\dot{b}$.

(b) Show that the loci $\dot{k} = 0$ and $\dot{b} = 0$ intersect only once at (b^*, k^*).

(c) Linearize the system around the non-Solow steady state (b^*, k^*). Show that this steady state is a saddle point.

(d) Precisely describe the region in (b, k) space in which the price of capital must be less than unity.

(e) Show that there are 3 and only 3 types of paths: (i) hyper-inflationary paths tending to the Solow balanced growth state, (ii) monetary paths tending to $(b^*, k^*) > 0$, (iii) hyper-deflationary paths that are revealed in finite time to be dis-equilibrium paths. Be precise. Carefully prove (iii).