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Economics 614: Macroeconomics II
Spring 2004
Cornell University
Problem Set #8
Due: Saturday, April 10, 2004

1 Leontief Production Function

$$C + Z = Y = \min[aK, bL]$$

(a) Write down the intensive production function and plot y on the vertical axis versus k on the horizontal axis.

(b) Using this production function, study the full dynamics of (i) the Solow model in continuous time, (ii) the Solow model in discrete time, (iii) the Ramsey-Cass-Koopmans model in continuous time.

2 Optimal Growth: Continuous Time

Consider the usual RCK optimal growth model except that the maximand is not smooth, viz.

$$\int_0^\infty u(c)e^{-\delta t}dt,$$

where $u(c) = -\infty$ for $c < \bar{c}$ and $u(c) = c$ for $c \geq \bar{c}$; $\bar{c} > 0$ can be interpreted as the minimum consumption to sustain life.

(a) Plot $u(c)$.

(b) Describe the full optimal growth dyanmics.

(c) Describe the turnpike property.

The marginal product of capital is 8% per year. Capital's share (of output) is 25%. The rate of depreciation of capital is 15% per year. The population is growing at 1% per year. What do you know about this economy?

3 Sunspots

2 individuals, $h = 1, 2$
2 states of nature, $s = \alpha, \beta$
1 indivisble good, $x_h \in \{0, 1\}$ for $h = 1, 2$

Let $\omega_1 = .3$ and $\omega_2 = .7$, so $\omega_1 + \omega_2 = 1$.

Fully describe all possible competitive equilibria. How do these depend on $(\pi(\alpha), \pi(\beta) = 1 - \pi(\alpha))$?

Replace the assumption of two states with the assumption that s is uniform on $[0, 1]$. Describe the set of competitive equilibria. What happens when moving from $s = \alpha, \beta$ to s uniform on [0,1]? What would be changed if the uniform distribution is replaced by a general continuous p.d.f. on [0,1]?

4 Government Debt Continued

$\log Y = .70 \log K + .30 \log L$
Savings out of disposable income, $s = .10$
Labor force growth, $n = .02$
Depreciation, $\mu = 0$
Continuous time

Plot:
(a) steady-state demand for consumption as a function of steady-state k and steady-state debt per head Δ
(b) steady-state supply of consumption as a function of steady-state k

Calculate:
(a) k^*, the golden-rule capital-labor ratio and the corresponding debt per head
(b) $\tilde{k}$, the maximum sustainable capital-labor ratio and the corresponding debt per head
(c) the Solow capital-labor ratio, k^0, corresponding to debt per head $\Delta = 0$
(d) the maximum sustainable debt per head, $\Delta^{\max}$, and the corresponding capital-labor ratio
(e) $\Delta^{\min}$, the maximum sustainable surplus per head and the corresponding capital-labor ratio.

Plot steady-state consumption per head vesus steady-state debt per head. How is the affected by the numerical parameters in this problem?