1. \( Y = AK^{0.5}L^{0.5} \), general price level is \( P \), nominal wage is \( W \), nominal capital cost is \( R \), the real wage \( w = W/P \). Markets are competitive. Firms are maximizing profits.

(a) Derive the firm’s labor demand function where \( K = 100 \) and \( A = 2 \).

(b) The labor supply function is \( L(w) = w^2 \). Draw the labor supply & demand curves and indicate the equilibrium wage on the graph.

(c) The technology has advanced from \( A = 2 \) to \( A = 4 \) where \( K = 100 \). Derive the labor demand function and equilibrium wage.

(d) There is an increase in capital from \( K = 100 \) to \( K = 400 \) where \( A = 2 \). Derive the labor demand function and equilibrium wage.

2. (Leontief Production Function) The production functions are given by

\[ F(K, L) = A \min \left( \frac{K}{a}, \frac{L}{b} \right) \]

(a) Derive the firm’s labor demand function where \( K = 100 \), \( A = 2 \) and \( a = b = 1 \).

(b) The labor supply function is \( L(w) = w^2 \). Draw the labor supply & demand curves and indicate the equilibrium wage on the graph.

(c) The technology has advanced from \( A = 2 \) to \( A = 4 \) where \( K = 100 \). Derive the labor demand function and equilibrium wage.

(d) There is an increase in capital from \( K = 100 \) to \( K = 400 \) where \( A = 2 \). Derive the labor demand function and equilibrium wage.

3. The production function \( F(K, L) \) is smoothly differentiable and exhibits constant returns-to-scale. Show that if capital’s share is \( 0 < \alpha < 1 \) is a constant fraction independent of the capital-labor ratio, then \( F \) is linear in logs (i.e. \( F \) is a Cobb-Douglas production function).
4. In CitrusLand, there are 2 goods: oranges and grapefruits. In period 1, the price of an orange is $10 and the price of a grapefruit is $1. Consumption of oranges is 2 and of grapefruits 80 in period 1. In period 2, the orange price has fallen to $1 and the grapefruit price has risen to $10; consumption of oranges has risen to 80, while consumption of grapefruit has fallen to 2.

(a) The CPI typically uses weights from the earlier period. What is the CPI value of CitrusLand?

(b) Show that there is an inflationary bias in the CitrusLand CPI. Hint: You might experiment with calculating an index using later period weights.