Problem Set # 5

1. **Uncertainty:** Let there be two states of nature, \( s = \alpha, \beta \). Formally establish the relationship between the equilibria from the contingent claims economy with equilibria from the spot-market-plus-Arrow-securities economy. **Notation:** Let \( x^i_h(s) \) be the holding by consumer \( h \) of commodity-\( i \)-deliverable-in-state-\( s \). Let the price of this contingent claim be \( p^i(s) \). Let \( b_h(s) \) be the holding by consumer \( h \) of money good only in state \( s \). Let the price of this security be \( p_b(s) \). Let \( \hat{x}^i_h(s) \) be consumption by \( h \) of \( i \) given the occurrence of \( s \). Let \( \hat{p}^i(s) \) be the spot market price of \( i \) given the occurrence of \( s \).

2. **Intertemporal:** Let there be two periods, \( t = 1, 2 \). Formally establish the relationship between the equilibria from the forward-market economy and the equilibria from the economy with spot markets. **Notation:** Let \( x^{i,t}_h \) be a forward purchase at price \( p^{i,t} \). Let \( \hat{x}^{i,t}_h \) be a spot purchase at price \( \hat{p}^{i,t} \). Let the money price be \( p^{m,t} \) and additions to money holdings be \( x^{m,t}_h \). **Hint 1:** Consumer \( h \) cannot plan to die a debtor. **Hint 2:** Find restrictions on \( p^{m,t} \) that follow from no-arbitrage in the money markets.

3. **Money-denominated taxes:**

   (3a) **One commodity:** Let \( \omega_1 = 7, \omega_2 = 15, \omega_3 = 4 \). Completely describe: (i) the set of bonafide taxes, \( (\tau_1, \tau_2, \tau_3) \), (ii) the set of normalized bonafide taxes, \( (\tau_1, \tau_2, \tau_3) \), and (iii) the set of equilibrium money prices. Which of these sets are convex?

   (3b) **l commodities:** Show that the above three sets are convex if the utility functions are linear in logarithms.

   (3c) **Convexity:** Are these sets convex in general? Write a one paragraph essay.

   (3d) **Connectivity:** Is the set \( P^m \) in general connected? Write a one paragraph essay.